Introduced by Chung in 1989, graph pebbling is a network optimization model for satisfying vertex demands with vertex supplies (called pebbles), with partial loss of pebbles in transit.

A configuration $C$ of size $|C| = p$ on a graph $G$ is a supply of $p$ pebbles on the vertices of $G$. Similarly, a target distribution $D$ of size $|D| = t$ on $G$ is a demand of $t$ pebbles on the vertices of $G$. A pebbling step from $u$ to an adjacent $v$ removes two pebbles from $u$ and places one of those pebbles on $v$; the other pebble vanishes as a toll.

We say that $C$ solves $D$ if $C$ can be converted via pebbling steps to a configuration $C^*$ such that $C^*(v) \geq D(v)$ for each vertex $v$. Generalized in 2005 for $t > 1$ by Crull, et al., the pebbling number, $\pi(G, D)$, of the target distribution $D$ is defined to be the smallest $m$ such that every configuration of size $m$ solves $D$.

The original case of $t = 1$ was proved in 2008 by Bunde, et al., to be $\Pi_2^P$-complete for general graphs, while Alcon, Gutierrez, and I conjectured in 2014 that the pebbling number of chordal graphs forbidding a particular subgraph called the Pyramid can be calculated in polynomial time. This work represents the farthest progress to date on this problem and is an important stepping stone toward tackling interval graphs.

In this talk I will present a number of new concepts and beneficial lemmas generated during the course of discovering a proof, many of which are quite general. We also strengthen the 2013 Target Conjecture of Herscovici, et al., and prove that it holds for powers of paths. This generalization is actually the key to the proof of the main result.

This is joint work with Liliana Alcón of the National University of La Plata, Argentina.