Can ETFs Increase Market Fragility? Effect of Information Linkages in ETF Markets*

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Abstract

We show how inter-market information linkages in ETFs can lead to market instability and herding. When underlying assets are hard-to-trade, informed trading may take place in the ETF. Underlying market makers, then, have an incentive to learn from ETF price. We demonstrate that this learning is imperfect: market makers pick up information unrelated to asset value along with pertinent information. This leads to propagation of shocks unrelated to fundamentals and causes market instability. Further, if market makers cannot instantaneously synchronize prices, inter-market learning can lead to herding, where speculators across markets trade identically, unhinged from fundamentals.

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1 Introduction

*Exchange-traded funds account for as much as a third of all publicly traded stocks. Or think of it this way: An ETF that tracks a basket of hard-to-trade emerging-market stocks or high-yield bonds will, on any day, attract more buy and sell orders than a bellwether like Microsoft or General Electric. Among bright ideas on Wall Street, this notion of promising investors instant liquidity in some of the most opaque corners of the global marketplace ranks with earlier innovations like securitization in consequence if not risk.*


What is the risk of introducing Exchange-Traded Funds (ETFs) on hard-to-trade assets? Do such ETFs amplify market volatility or do they act as shock absorbers, introducing an extra layer of liquidity? Shock absorbers or not, what is the mechanism by which they affect market trading? Can these ETFs lead to trading frenzies in which many speculators rush to trade on the same market signal causing large price dislocations? What can regulators do to promote dampening effects of ETFs, if any, over amplifying effects? Questions like these have become increasingly important in recent years as interest in this instrument has exploded.\(^1\) ETFs today are often the preferred vehicle for access to assets that have limited participation or liquidity otherwise, making them a systemically important cog that may move risks across markets.\(^2\) Despite widespread interest among practitioners and regulators, academic research—especially theoretical work—on these market impacts of ETFs is scant.

In this paper we address this gap in the literature by investigating the risks and

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benefits of ETFs that track hard-to-trade assets. We are particularly interested in understanding when trading in ETFs may lead to greater market fragility. The term fragility has been used in the literature for many different kinds of market vulnerabilities. In this paper, we use fragility to refer to two specific phenomena: (i) market instability, driven by propagation of shocks unrelated to fundamental value of an asset, and, (ii) rational herding, wherein all market speculators trade in the same direction, on the same market signals, unhinged from asset fundamentals. Our analysis identifies a feedback channel in which herding and instability can arise due to the presence of ETFs.

Specifically, we develop a tractable model of ETF trading that features learning and feedback effects from the ETF to the underlying asset markets and vice versa. In our model the ETF tracks the weighted average of a basket of underlying assets, and markets are organized as conventional Kyle-style auctions. An important feature of ETF markets is the creation/redemption process which serves to link underlying and ETF prices. In practice, such perfect synchronization is not automatic, particularly when market participants may not have synchronous or symmetric access to both the ETF and the underlying assets. Our focus is on ETF settings where the underlying assets are hard-to-trade, implying that ETF arbitrageurs cannot immediately trade away price differences. Hard-to-trade underlying markets also imply significant price discovery in ETFs, because market participants cannot easily access underlying assets. In such scenarios, ETF prices can serve as a source of information for market makers in underlying assets, and vice versa, setting the stage for important feedback effects between prices that drive our key results.

The 'hard-to-trade' moniker applies to a variety of assets. For instance, trade in some foreign sovereign stocks requires licenses that ETF traders often do not have, rendering the underlying assets out of bounds for speculators except as part of country ETFs. Similarly, in certain commodity ETFs, trade in the underlying requires the capacity to carry
the physical asset, precluding ETF speculators from also participating in the underlying. For many bond ETFs on the other hand, underlying markets are still over-the-counter and illiquid, and trade may be difficult (and expensive).\textsuperscript{3} At the same time, speculators trading in such underlying asset markets may be specialized, with little incentive to trade the full ETF basket.\textsuperscript{4} Lack of symmetric access may also arise due to asynchronous trading. Trading hours for ETFs and underlying assets do not overlap in many ETF markets, and in turbulent conditions the underlying market may even close while the ETF remains open (a case in point being Greek ETFs in summer 2015).

It is important to recognize that hard-to-trade or easy-to-trade is not a water-tight classification. Various assets, that may ordinarily be easy to buy and sell, become hard to trade under special circumstances. For example, on August 24, 2015, the US markets witnessed over 1200 trading halts (limit ups and limit downs) in the opening hour of trade, rendering many of the biggest blue-chip stocks largely inaccessible.\textsuperscript{5} Similarly, in April 2014, trade in the Japanese sovereign bond market (the world’s second largest sovereign debt market) dried up to a large extent due to the massive asset purchase program implemented by the Bank of Japan.\textsuperscript{6} In all such scenarios—where trade disruptions may have temporarily diminished access to underlying assets leading to price discovery in ETFs—our analysis becomes relevant.

\textsuperscript{3}The ETF tracking website “ETF Database” (etfdb.com) estimates that, as of October 2016, about $150 billion is parked in ETFs on emerging market equities and bonds, over $10 billion in foreign small and mid-cap equities, about $60 billion in precious metal ETFs (with underlying being the physical gold/diamond/platinum), about $50 billion in junk bond ETFs and about $60 billion in real estate ETFs. Together, these categories constitute over 10\% of the US ETF universe. The actual size of the ETF on hard-to-trade assets subuniverse could be much larger because, as we describe in the next paragraph, many so-called easy to trade, liquid assets face periodic bouts of illiquidity and trade disruptions.

\textsuperscript{4}An alternative explanation as to why some speculators may stick with ETFs, while others trade in only certain specific underlying stocks, comes from the behavioral literature that explores cognitive biases like familiarity and home bias.

\textsuperscript{5}Refer to the SEC research note titled “Equity Market Volatility on August 24, 2015,” available at https://www.sec.gov/marketstructure/research/equity_market_volatility.pdf

Market instability arises in such settings because an underlying market maker, when learning from ETF prices, cannot perfectly distinguish between price changes caused by factors pertinent to his asset, and other factors, irrelevant to him. This leads to a situation where idiosyncratic shocks pertinent to one asset begin to affect the price of another independent asset—through the ETF price channel—thus causing market instability. We show that ETF markets bring both benefits and costs for underlying asset markets. At the level of the aggregate basket, ETF trading helps move underlying prices closer to fundamental value. Yet at the level of individual assets, it may lead to persistent distortions from fundamentals. Assets with high beta and high weightage in the ETF are especially vulnerable to such distortions.

We also show how herding can arise in the ETF ecosystem. When ETF market makers cannot instantaneously synchronize their price with underlying prices through the arbitrage mechanism, market makers in the ETF and underlying markets set initial clearing prices based on order flow in their own markets, and then revise them as they see prices in other markets evolve. This staggered information flow offers speculators an opportunity to use short-horizon strategies, closing out positions without waiting for the liquidation value to realize, if they can correctly guess the information flow from other markets. This can happen if all speculators possess a common signal on which they can co-ordinate their trade—and the systematic factor signal, which all speculators tend to track because it affects all asset prices, fits the criterion. Indeed, we show that there exists an equilibrium where speculators across the ETF ecosystem end up herding on the systematic factor signal. This herding phenomenon is reminiscent of a Keynesian beauty contest: speculators raise the weight on the systematic factor, not because it affects the liquidation value, but because other speculators do the same.

Though we focus exclusively on ETF markets, an interesting question is the extent to which our results generalize to other asset settings. In this paper, we require our asset to
satisfy four criteria: (1) it must be a “basket” asset, (2) there must be significant independent price discovery in the basket, (3) there must be limits to arbitrage that prevent basket and underlying prices from instantly synchronizing, and (4) for herding, at least one of the factors that determine price must be common to all assets in the basket. The first condition is necessary for unrelated shocks to enter our ecosystem, while the second and third conditions make sure there is inter-market learning.\footnote{As already emphasized, the “hard-to-trade underlying” restriction on our ETF set ensures that the second and third conditions are naturally satisfied by our ETFs.} Herding is essentially a co-ordination phenomenon and a common factor signal, our fourth condition, serves as a co-ordination device. Technically, any basket asset that satisfies the conditions we impose on ETFs in this paper is capable of exhibiting the phenomena we describe. In reality, not too many assets other than ETFs satisfy all the conditions. For instance, index funds do not witness independent price discovery because they are not traded on the exchange. Important index futures, on the other hand, rarely witness significant breakdown of the arbitrage mechanism. Furthermore, futures prices are more forward looking than underlying stock prices rendering co-ordination on a common factor significantly harder.

That ETFs could exert independent effects on market behavior reflects the underlying enigma posed by these securities. As derivatives, ETF prices should be determined by the values of their underlying assets. Yet, in many cases, the trading volume (and liquidity) of the ETF far exceeds that of the underlying asset markets, making these the preferred vehicle of trading interest. And with that trading interest comes the possibility that informed trading (and price discovery) now takes place in the ETF. When this occurs, it is akin to the “tail wagging the dog”, in that the ETF price changes the underlying prices rather than the underlying prices changing the ETF. As events like the market dislocation on August 24, 2015 made clear, ETFs are no longer simple appendages to the market, but rather are now capable of affecting markets in their own right.

The paper is organized as follows. Section 2 provides a brief overview to the literature.
Section 3 then introduces our basic model. In Section 4 we study ETF markets where the underlying is hard to trade, resulting in informed speculation occurring primarily in the ETF. In such markets, we show that in equilibrium, shocks unrelated to fundamentals may propagate from asset to asset through the ETF channel. In Section 5 we allow informed speculation in both underlying markets and in the ETF. We show that such markets admit a herding equilibrium where all speculators use only the systematic factor signal to determine their order size and asset prices become unhinged from fundamentals. In Section 6 we use data from Greek ETF trading during the financial crisis in summer 2015 to illustrate many of the phenomena we describe in the paper. Section 7 discusses policy implications and concludes. All proofs are in the Appendix.

2 Literature Review

Our paper is related to a growing and diverse body of literature. One such area is models with feedback effects and strategic complementarities in financial markets (see Hirshleifer, Subrahmanyam and Titman (1994), Barlevy and Veronesi (2003), Veldcamp (2006), Ganguli and Yang (2009), Amador and Weill (2010), Garcia and Strobl (2011), Goldstein, Ozdenoren and Yuan (2011), Goldstein, Ozdenoren and Yuan (2013), Hassan and Mertens (2014)). Most of these papers focus on strategic complementarities that arise in information acquisition and interpretation, but none to our knowledge have focused on the particular complications introduced by ETFs. Two papers are particularly relevant for our analysis here. Cespa and Foucault (2014), using a rational expectations framework, show how liquidity shocks in one asset can spill over to other assets. Our analysis focusses on information linkages and asset price co-movements, not liquidity, but both models suggest the important role played by cross-asset learning in affecting market behavior. The herding portion of our model is related to the model in Froot, Sharfstein
and Stein (1992), though the driving mechanisms differ between the two papers. In their model herding arises because of execution uncertainty: half the orders sent by an informed speculator execute in the first period, while the other half execute in the second period. Thus if a speculator wants to close out his position after the second period, he imitates other speculators. In our model, on the other hand, the reason for herding is the information linkage between the markets that arises due to the way market makers learn.

Our work is also related to market microstructure research looking at the impact of an index on trading activity. Subrahmanyam (1991) examines the optimal strategy for discretionary liquidity traders when they can trade in both the index and the underlying securities, and shows that adverse selection costs are typically lower in indexes. Introduction of an index, therefore, reduces liquidity in underlying securities because liquidity traders find the index more attractive. Gorton and Pennacchi (1993) show that when prices are not fully revealing, the return on composite securities cannot be replicated by holding the underlying individual assets when investors have immediate needs to trade. Like Subrahmanyam (1991), they show that an index can improve the welfare of uninformed traders. The focus of our paper is quite different from these papers, however, as we look at how inter-market information linkages in ETF markets can cause instability and lead to herding.

There is a growing empirical literature looking at the impact of equity ETFs on the stock market. Ben-David, Franzoni and Moussawi (2014) and Krause, Ehsani and Lien (2014) find that ETFs increase the volatility of underlying assets, and Ben-David, Franzoni and Moussawi (2014) further show that this is not accompanied by increased price discovery at the stock level. Da and Shive (2015) find that ETFs contribute to equity return co-movement, an increase they argue is not due to fundamental factors. Israeli, Lee and Sridharan (2016) find that increased ETF ownership is accompanied by increased bid-
ask spreads, decreased pricing efficiency, and increased co-movement of the underlying stocks. The findings in these papers are broadly consistent with our model. Glosten, Nallareddy and Zhou (2016), using quarterly data and an accounting-based measure of informativeness, argue that greater ETF holdings of a stock increase the informational efficiency of small stocks and stocks with imperfect equity market competition. We show that ETFs can decrease the informational efficiency of the underlying security, but our focus is on shorter horizons, where limits to arbitrage are relevant. Whether ETFs increase informational efficiency over long horizons is not a focus of our analysis.

Finally, another related literature involves the limits to arbitrage (see for example Shleifer and Vishny (1997); Gromb and Vayanos (2010)). This literature has largely focused on understanding how the capital and risk required in real world arbitrage can result in asset prices diverging from true values. Because an ETF is a composite security, in principle its price at any time is simply the summation of the underlying component prices. Deviations from these prices are supposed to be “instantaneously corrected” by an arbitrage process that involves the creation and redemption of ETF shares. In practice, such perfect synchronization is not automatic, particularly when limited (or asynchronous) access (or simply greater liquidity) makes the ETF a preferred venue for informed trading. Malamud (2015) analyzes the impact of the ETF redemption process in a model with symmetric information, and trading done only for risk-sharing purposes. His analysis shows that arbitrageurs face limits to arbitrage due to execution risk, arising in part from risk aversion. In his setup, the rigidities in the redemption process can introduce new dynamics into ETF prices due to inventory considerations. Our analysis here is based on risk neutral market participants, asymmetric information and the learning interaction between ETFs and underlying securities. Our focus on ETFs based on hard to trade assets underscores how market access and information flows can also create natural impediments to arbitrage—and the potential for market fragility.
3 A Model of ETF Trading

To study market instability that arises through propagation of shocks unrelated to asset fundamentals, we develop a model of ETF trading based on the classic Kyle (1985) setup. In the real world, an ETF originates when an issuer of ETF (ETF sponsor) designates chosen market participants as ETF market makers (authorized participants). Authorized participants have a special agreement with the ETF issuer: they can create/redeem ETF shares by either delivering the constituents of the ETF (called an “in-kind” transaction), or by offering the net asset value equivalent of cash if the underlying assets are easily tradable (called an “in-cash” transaction). In an in-cash transaction, the ETF sponsor obtains the replicating basket himself. Creation/redemption follows a pre-defined procedure specified in the authorized participant contract: it happens in pre-defined large blocks (often, 50,000 ETF shares or higher), at designated times (usually, end-of-day) and designated prices (usually, end-of-day closing prices or opening prices next day). While most ETFs historically allowed only in-kind creations/redemptions, nowadays many allow cash redemptions, or a mix of both.\footnote{For more details on the creation/redemption process refer Shreck, M. and S. Antoniewicz (2012, January 9). ETF Basics: The Creation and Redemption Process and Why It Matters. Retrieved March 15, 2016, from https://www.ici.org/viewpoints/view_12_etfbasics_creation.} When ETF and underlying asset markets are liquid—and trade synchronously—authorized participants can arbitrage away price differences between the underlying basket and ETF ensuring that they move largely in-step. For instance, if the ETF is trading at a premium, authorized participants can sell short the ETF while simultaneously buying the underlying securities. At the end of the day authorized participants may deliver the basket of securities to the sponsor, in exchange for ETF shares, thus closing out the position for a profit.

\footnote{In this paper we focus on “physical” ETFs where the sponsor physically holds the replicating basket of assets. There is another category of ETFs, the so-called “synthetic” ETFs, where the sponsor uses derivatives such as swaps to track an underlying index.}
positions of authorized participants—and thus their participation in ETF markets if they care for inventory risk.\textsuperscript{10} In this paper we deliberately abstract away from such inventory considerations by assuming that all market participants are risk-neutral—instead, we focus is on information linkages among markets. The underlying markets in our model are hard-to-trade, thus authorized participants cannot immediately obtain replicating portfolios of underlying assets after trading an ETF. Furthermore, due to the barriers to trade, order flows in the ETF and underlying markets have different sources. In our risk-neutral world with asymmetric information, this implies that the key driver of the price adjustment process is market participant expectations about how asset prices should evolve over time as more information gets incorporated into the prices through the process of trading.

\textbf{Asset Value}

We begin by setting up a model in which there is one exchange traded fund (denoted by \( e \)) tracking the weighted average of \( N \) underlying assets. The initial value of security \( i \) \((i = 1, \ldots, N)\), \( P_{i,0} \), is public knowledge. The liquidation value of the asset, \( v_i \), is given by

\[
P_{i,0} + b_i \gamma + \epsilon_i, \quad i = 1, \ldots, N,
\]

where \( \gamma, \epsilon_1, \ldots, \epsilon_N \) are all mutually independent normally distributed random variables, each with mean zero. Consistent with “factor model” representation of security prices, shocks to the asset value may be decomposed into a systematic (or common) factor component, \( \gamma \), and an idiosyncratic component, \( \epsilon_i \), with \( b_i \) denoting the factor loading. When it simplifies calculations without loss of generality, we assume \( \text{var} (\epsilon_i) = \text{var} (\epsilon_j) = \text{var} (\epsilon) \forall i, j \in \{1, \ldots, N\} \), i.e., the variances of the idiosyncratic components are equal.

\textsuperscript{10}For example, block size requirements for creation/redemption imply that authorized participants may have to carry inventory on their books for extended periods of time.
The value of the ETF is simply the weighted average of the underlying asset prices. Thus the liquidation value of the ETF is

\[ \sum_{i=1}^{N} w_i P_{i,0} + \sum_{i=1}^{N} w_i b_i \gamma + \sum_{i=1}^{N} w_i \epsilon_i, \]  

(2)

where \( w_i \) is the weight of asset \( i \) in the basket. For simplicity, unless otherwise stated, we assume \( P_{i,0} = 0 \).

**Market Participants and their Information**

The ETF and assets are traded simultaneously in separate markets. Each market is organized as a Kyle (1985) type auction with a designated market maker. All traders in the model are risk-neutral. There is one *informed speculator* who trades only in the ETF market. This speculator receives \( N + 1 \) signals: \( N \) signals about the \( N \) idiosyncratic factors, and one signal about the systematic factor. We also have one informed speculator in each of the \( N \) underlying asset markets. This speculator receives a signal about the idiosyncratic factor affecting his specific market, and a signal about the systematic factor. We analyze a model with speculators in both ETFs and underlying markets in Section 5, but as a useful preliminary, in Section 4, we consider a world in which informed speculators are only active in the ETFs. For simplicity, we assume that signals received by speculators in all the markets—ETF and underlying—have no noise: the speculator trading in market \( i \) observes \( \epsilon_i \) and \( \gamma \), and so does the ETF speculator.

There are \( N + 1 \) *market makers* in the model: one for each underlying asset market and an authorized participant for the ETF market.\(^{11}\) Like in Kyle (1985), (unmodeled) competition is assumed to drive their profits to zero, so they clear markets at expected value. As is standard in such models, there are *liquidity traders* in the ETF and underlying markets who are assumed to have exogenous reasons for trade. Liquidity traders in

\(^{11}\)We use the terms authorized participant and ETF market maker interchangeably.
market \( i \) place an order of \( z_i \sim \mathcal{N}(0, \text{var}(z_i)) \); in the ETF market, the liquidity order is \( z_e \sim \mathcal{N}(0, \text{var}(z_e)) \). We assume that the variance of liquidity orders in the markets are identical, i.e., \( \text{var}(z_e) = \text{var}(z_i) = \text{var}(z_j) = \text{var}(z) \forall i, j \in \{1, ..., N\} \).

**Timing of Trade**

There are three dates, \( t = 1, 2, 3 \). On date 1, informed speculators in the ETF and underlying markets trade in their respective markets according to their information. On date 2, market makers in the underlying markets update their prices after observing the date 1 ETF price, and the authorized participant updates the ETF price after observing the date 1 underlying market prices. On date 3, the authorized participant buys/sells the underlying assets and creates/redeems ETF shares with the sponsor to close out his position in an in-kind transaction. In an in-cash transaction, the authorized participant offers cash, equivalent to NAV of the ETF. The value of underlying assets, on date 3, is assumed to be the liquidation value. (Figures 1 and 4, in Sections 4 and 5 respectively, illustrate the timeline graphically.)

The timeline described above, while it captures the essentials of the trading process, is a simplification of reality. When underlying assets are hard to trade, the authorized participant may not be able to obtain the replicating portfolio of underlying assets all at once—as we assume—nor is it necessary that the liquidation value of all assets realize at the same time, when the authorized participant is closing out his position. But these abstractions simplify the solution of our model greatly, without changing the (qualitative) results of the paper. If, for example, authorized participant trade in underlying markets were staggered, creation/redeemption would be postponed till the entire replicating portfolio were obtained. Yet we would still obtain similar results since the learning problems would stay essentially unchanged. Similarly, the liquidation value moniker could be applied to whatever asset values prevail when the creation/redeemption process happens.
The objective of the ETF speculator is to choose an order size, \( x_e \), that satisfies

\[
x_e = \text{arg} \max_{x_e} \mathbb{E} \left( x'_e \left( \sum_{j=1}^{N} w_j (\epsilon_j + b_j \gamma) - P_{e,1} \right) \right),
\]

where \( P_{e,1} \) denotes the ETF price on date 1. Similarly, the objective of a speculator in underlying market \( i \) is to choose

\[
x_i = \text{arg} \max_{x_i} \mathbb{E} \left( x'_i (\epsilon_i + b_i \gamma - P_{i,1}) \right).
\]

The total order flow in the ETF market is denoted by \( q_e = x_e + z_e \). Similarly, in underlying market \( i \), the total order flow is \( q_i = x_i + z_i \).

**Learning and Equilibrium**

The model developed here involves \( N + 1 \) securities, \( N + 1 \) informed speculators, and \( N + 1 \) market makers. In principle, a complete solution to this model could involve a complex equilibrium in which underlying market makers learn not only from their own order flow, the price movements of every other underlying asset, and the price movements of the ETF, but also from the lack of order flows and price movements in their own and other securities. Such a complicated learning problem is intractable. In the following analysis, we focus on more tractable learning scenarios. In the next section, we first characterize how a market maker would learn from the ETF and impound that information in the underlying security. The subsequent section then allows learning from both the ETF and the own security and focuses on equilibria that result in herding.
4 Informed Speculators in ETF

In this section, we analyze equilibrium when the underlying asset markets are not easily accessible to informed speculators. From a modeling perspective, this implies two assumptions: (i) price discovery happens in the ETF, and, (ii) the creation/redemption process, and thus the arbitrage mechanism between the ETF and underlying assets, is disrupted till date 3. As noted in the introduction, a number of high yield bond, commodity and country ETF markets possess the inaccessibility characteristic that we model here, and so too would any setting in which the ETF trades when the underlying market is not open. The practical import of this assumption is to restrict our attention to informed speculation happening only in the ETF. For analytical simplicity, we assume complete inaccessibility—and thus no trade in underlying asset markets—but our results go through, qualitatively, when the inaccessibility is partial—as long as there is substantial price-discovery in the ETF that forces underlying market makers to learn from ETF price changes. In the next section we analyze a setting with price discovery in both the underlying and the ETF.

Figure 1 presents the timeline for this setup. Underlying market makers track ETF price changes for information about their own assets. With no informed trading in asset
markets, $P_{i,1} = P_{i,0}$, and since ETF market makers have nothing to learn from underlying markets, $P_{e,2} = P_{e,1}$. As is standard in the literature, we look at symmetric linear equilibrium strategies for informed speculators. Each market participant conjectures the strategies of all other participants; in equilibrium, the conjectures are consistent.

An underlying market maker, on seeing a change in ETF price, can infer the order flow as $q_e = (P_{e,1} - P_{e,0}) / \lambda_e$, where $\lambda_e$ denotes the price impact factor in the ETF market determined in equilibrium. From this order flow, the underlying market maker tries to discern information pertinent to his asset. As a Bayesian, he therefore revises the price in his own market to

$$P_{i,2} = P_{i,1} + \lambda_e q_e = P_{i,1} + \frac{\text{cov}(\epsilon_i + b_i \gamma, q_e)}{\text{var}(q_e)} q_e,$$

(5)

where $\lambda_{ei}$ denotes the impact of the ETF price change on underlying asset $i$.

An ETF speculator takes into account the impact of his trade on the ETF price before placing an order; hence the clearing price offered by the market maker in the ETF aggregates the information on the systematic factor and all idiosyncratic factors. But this, in turn, implies that the order flow inferred by an underlying asset market maker has information pertinent to the asset mixed not just with random noise, but also with systematic information related to other underlying assets. In other words, if $\nu_{ej}$ and $\theta_e$ denote the optimal weights that an ETF speculator places on the idiosyncratic $j$ and systematic factor signals respectively in equilibrium, the order flow inferred by underlying market maker $i$ is

$$q_e = z_e + \sum_{j=1}^{N} \nu_{ej} \epsilon_j + \theta_e \gamma.$$

(6)

In equation (6), $\nu_{ei} \epsilon_i + \theta_e \gamma$ is the only component of the order flow that is pertinent for underlying market maker $i$, the rest of it obfuscates the information. Substituting the value for $q_e$ in equation (5) above, we obtain the impact of the ETF price adjustment on
underlying asset market $i$:

$$P_{i,2} = P_{i,1} + \frac{\nu_i \text{var} (\epsilon_i) + \theta_b \text{var} (\gamma)}{\sum_{j=1}^{N} \nu_j \text{var} (\epsilon_j) + \theta_b \text{var} (\gamma) + \text{var} (z_e)} \left[ z_e + \sum_{j=1}^{N} \nu_j \epsilon_j + \theta \epsilon \gamma \right].$$

(7)

Solving for parameters $\nu_{ij}$, $\theta_e$, $\lambda_e$ and $\lambda_e$ gives us the proposition below.

**Proposition 1.** The equilibrium price set by the authorized participant in the ETF market is

$$P_{e,1} = \lambda_e q_e,$$

(8)

the equilibrium price set by the market maker in underlying market $i$, $i = 1, ..., N$, is

$$P_{i,2} = \lambda_e q_e,$$

(9)

and the optimal order size of the informed ETF speculator is

$$x_e = \sum_{j=1}^{N} \nu_{ij} \epsilon_{ij} + \theta \epsilon \gamma,$$

where

$$\nu_{ij} = \frac{w_j}{2\lambda_e}, \quad \theta_e = \frac{\sum_{j=1}^{N} w_j b_j}{2\lambda_e},$$

(10)

$$\lambda_e = \sqrt{\frac{\left(\sum_{j=1}^{N} w_j^2\right) \text{var} (\epsilon) + \left(\sum_{j=1}^{N} b_j w_j\right)^2 \text{var} (\gamma)}{4 \text{var} (z)},}$$

(11)

and $\lambda_e$ is

$$\lambda_e = \frac{\lambda_e w_i \text{var} (\epsilon) + \lambda_e b_i \left(\sum_{j=1}^{N} b_j w_j\right) \text{var} (\gamma)}{\left(\sum_{j=1}^{N} w_j^2\right) \text{var} (\epsilon) + \left(\sum_{j=1}^{N} b_j w_j\right)^2 \text{var} (\gamma)}.$$

(12)

Proposition 1 illustrates how ETFs tracking hard-to-trade assets may lead to *market instability*. Recall that market instability, in our context, refers to the propagation of unrelated shocks across assets. By unrelated, we mean shocks that are independent of factors that determine the fundamental value of the asset. As Proposition 1 shows,
the underlying market maker in asset \( i \) is now influenced by information related to the collection of assets.

A novel feature of information transmission through inferred ETF order flow is that it leads to underlying markets getting “coupled”. Observe that the only source of information for market makers in the underlying asset markets is informed trading in the ETF, and equation (5) above describes how they learn from the ETF price. Coupling, in this case, happens through two channels. The first channel is the price impact factor, \( \lambda_{ei} \), in equation (5). Equation (12) shows that the price impact factor in market \( i \) is affected by the weights, betas, and variance of idiosyncratic factor of other assets in the ETF, as well as the number of assets in the ETF—even though these variables are not related to asset \( i \)'s liquidation value. The second channel is the order flow variable in equation (6): from the aggregate order flow that he infers, an underlying market maker has no way of distinguishing shocks pertinent to his asset, from irrelevant shocks to idiosyncratic factors of other assets. It is this inability to discriminate that allows unrelated shocks to affect underlying asset prices. Our model allows a precise characterization of the transmission.

**Proposition 2.** (Market instability) A shock of \( \eta_j \) to the idiosyncratic component of asset \( j \) leads to a shock of

\[
\frac{w_i w_j \lambda_e \text{var}(\epsilon) + w_j \lambda_{eb} \left( \sum_{j=1}^{N} w_j b_j \right) \text{var}(\gamma)}{2 \left( \sum_{j=1}^{N} w_j^2 \right) \text{var}(\epsilon) + 2 \left( \sum_{j=1}^{N} w_j b_j \right)^2 \text{var}(\gamma)} \eta_j
\]

\[\text{(13)}\]

to \( P_{i,2} \), the price of asset \( i \).

Proposition 2 demonstrates an important way in which ETFs affect overall market stability. Equation (13) shows that, unlike other common financial instruments, ETFs can act as conduits for transmission of risks across the market ecosystem — in this sense, therefore, ETFs make the ecosystem more coupled. Given that there are over 6100 ETFs now available to investors globally, the influence of these instruments on market fragility
In both figures, we model an ETF with 20 underlying assets. For the figure on the left, the weight of each asset is 5%, and the asset betas range between 0 and 1.5. For the figure on the right, the weight of each asset ranges between 0 and 10%, and the asset betas are all 1. For both figures, the variance of the systematic factor $\text{var}(\gamma)$, the variance of the idiosyncratic factors $\text{var}(\epsilon)$, and the variance of noise trade $\text{var}(z)$, are 1 unit. The unrelated idiosyncratic shock $\eta_j$ is also assumed to be 1 unit.


Equation (13) allows us to delineate important determinants of shock propagation.

**Corollary 1.** (Proposition 2) *Ceterus paribus, the impact of an unrelated shock is higher for an asset with higher absolute beta, when all asset betas in the ETF have the same sign.*

Corollary 1 follows directly from equation (13). To understand it intuitively, note that since all speculators use identical systematic factor signals to decide their order size, underlying market makers expect to find good information about the systematic factor in the inferred ETF order flow. Higher beta implies that the systematic factor has a higher relative weightage for the value of the asset, thus a market maker gives higher importance to information in ETF order flow. This, in turn, implies a greater vulnerability to unrelated shocks.

**Corollary 2.** (Proposition 2) *Ceterus paribus, the impact of an unrelated shock is higher*
for an asset with higher weight in the ETF, when all asset betas in the ETF have the same sign.

Corollary 2 reflects the fact that when the ETF speculator has information about an asset that has a higher weight in the ETF, he trades on the basis of that signal relatively more, because it gives him a greater relative informational advantage. Consequently, underlying market makers in those markets learn more from the ETF price adjustment and are thus more susceptible to unrelated shocks.

ETF markets therefore bring both benefits and costs for underlying asset markets. The cost is that irrelevant information, blended with pertinent information, now affects prices; the benefit is the access to more information. In the classic setup of Kyle (1985), informativeness of prices is measured by the change in variance of the market maker’s value distribution for the asset after a round of trading. Kyle shows that the posterior variance is one-half of the prior variance, and interprets this as revelation of half the information of a speculator in each round. In the following proposition, we show that in our model too, information in ETF order flow brings down the variance for underlying market makers.

**Proposition 3.** The posterior variance of underlying market maker i’s distribution for the asset value, after observing the change in ETF price, is

\[
\frac{\left(2 \sum_{j=1}^{N} w_j^2 - w_i^2\right) \text{var}^2(\epsilon) + b_i^2 \left(\sum_{j=1}^{N} b_j w_j\right)^2 \text{var}^2(\gamma)}{2 \left(\sum_{j=1}^{N} w_j^2\right) \text{var}(\epsilon) + 2 \left(\sum_{j=1}^{N} w_j b_j\right)^2 \text{var}(\gamma)}
+ \frac{\left(b_i^2 \sum_{j=1}^{N} w_j^2 - b_i \left(\sum_{j=1}^{N} b_j w_j\right) + \left(\sum_{j=1}^{N} b_j w_j\right)^2\right) \text{var}(\epsilon) \text{var}(\gamma)}{\left(\sum_{j=1}^{N} w_j^2\right) \text{var}(\epsilon) + \left(\sum_{j=1}^{N} w_j b_j\right)^2 \text{var}(\gamma)}. \tag{14}
\]

**Corollary.** (Proposition 3) Underlying market maker i’s posterior variance for the asset value distribution, after observing the change in ETF price, is lower than his prior
variance.

It is important to place Proposition 3 in the right perspective. The Corollary shows that market makers are less uncertain about the value of the asset after they learn from the ETF order flow. This indicates that speculators have conveyed information through the trading process. In the classic Kyle (1985) model, this implies that, on average, prices have moved closer to the true value. In other words, if the trading game were repeated a sufficiently large number of times in Kyle (1985), prices will have converged to the true asset value. As Figure 3 illustrates, in our model, the implication is more subtle.

At the level of the aggregate underlying basket, the Kyle implication holds true. Like in Kyle (1985), this follows directly from the random nature of liquidity trading. By the weak law of large numbers, \( \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} (z_e)_n = 0 \), and thus after sufficiently large number of rounds, speculator order flow gets separated perfectly from liquidity order flow; hence aggregate price of the basket converges to the true value. Unlike Kyle (1985), however, the speculator order flow is not homogeneously informative. The ETF speculator has information about all underlying assets, and the aggregate order flow represents the totality of the information held by the speculator. Unlike the random nature of liquidity trading, order flow from the ETF speculator has a systematic bias due to the information driving the trade. Consequently, it is impossible for an underlying market maker to distinguish perfectly information pertinent only to his specific asset—from the ETF price—however many times the game be repeated. This, in essence, represents one of the central dichotomies of the effect of ETFs: at the level of the aggregate basket, prices are better informed, but at level of individual prices, there can be persistent distortions from fundamentals.
Figure 3: Illustrative plots showing that while the value distribution for the ETF basket converges to the true value as the game is repeated, an underlying market maker’s value distribution may not converge

(a) Initial mean of authorized participant’s value distribution for ETF basket=100,
Actual value of ETF basket=350,
The ETF authorized participant’s value distribution converges to the actual value as the game is repeated sufficient number of times.

(b) Initial mean of underlying market maker’s value distribution for asset=100,
Actual value of asset=250,
The underlying market maker’s value distribution does not converge to the actual value even if the game is repeated a number of times.

The simulations were run using 5 assets, i.e. \( N=5 \), with each asset having equal weight in the ETF. The actual value of the assets were taken to be 150, 250, 350, 450 and 550. The plots above represent one of the many possible paths. In the case of the ETF, all paths converge to 350.
5 Underlying Markets with Speculators

Having analyzed the inference problem from the ETF when underlying markets have no trade, we now turn to a model in which some speculation takes place in both the ETF and underlying markets. From a modeling perspective, this implies that: (i) there is partial price discovery in both ETF and underlying assets, and (ii) the creation/redemption process continues to be disrupted till date 3. As described in the model section, each underlying market has one informed speculator, as well as there being an informed speculator trading only the ETF. Figure 4 gives the timeline for the setup. Thus \( P_{i,1} \) and \( P_{e,2} \) no longer need be equal to \( P_{i,0} \) and \( P_{e,1} \) respectively.

Even though there is some trading in underlying markets now, we continue to assume that the ETF speculator trades only in the ETF. A reason for this is trading frictions that hamper easy access to such markets—even when they are technically open for trade.
For instance, O’Hara, Wang and Zhou (2016) suggest that in corporate bond markets, trading relationships and networks make a big difference to the terms of trade for market participants. Similar barriers to trade are common also in commodity, real estate and foreign asset markets. At the same time, specialized speculators, who are often long time participants in these underlying markets, have little incentive to trade the full ETF basket. For one, such speculators tend to receive beneficial terms of trade in the underlying market due to their long association. For another, since these speculators specialize in a particular underlying market, their relative informational advantage is usually higher when trading in the particular asset.\(^1\) In our model, we therefore assume that underlying market speculators do not trade in the ETF.\(^2\)

Though the institutional context is very different, some features of the equilibrium that we obtain in this section are similar to the herding equilibrium discussed in Froot, Sharfstein and Stein (1992). When there is informed speculation in both ETF and underlying markets, speculators may have profitable “short-horizon” trading strategies. In our context, short-horizon trading strategies for speculators are those where they exit the market on intermediate date 2, without waiting for the liquidation value of the assets to realize. This can be profitable for speculators because the market makers learn in stages: initially they set prices to reflect information in just their own market; at a later stage, when they see prices in other markets, market makers revise their prices to reflect new information. If speculators can foresee correctly the price changes in other markets, the

\(^1\)Technically, in any underlying market \(i\), a specialized informed speculator knows all the components that make up the final asset price, \(\epsilon_i + b_i\gamma\). In contrast, if this speculator trades in the ETF, all idiosyncratic factors other than the one that affects his market, i.e. \(\epsilon_j, j \neq i\), are unknown to him. So even when he does not obtain special terms of trade in the underlying market, the only scenario where this speculator ex-ante might prefer trading in the ETF is when the weighted average of the asset betas \(\sum w_i b_i\) is sufficiently high, rendering the knowledge of the common factor \(\gamma\) especially valuable in determining the final value of the ETF. If one takes into account the additional gains from the beneficial terms of trade that speculators often receive in over-the-counter markets, even this scenario seems unlikely.

\(^2\)These assumptions simplify our calculations considerably, but our herding results go through even when ETF speculators have partial access to underlying markets, and vice-versa.
intermediate revision of prices by market makers offers them an opportunity to close out positions profitably, without waiting for realization of liquidation value, and we analyze below the conditions under which this may happen. Following Froot, Sharfstein and Stein (1992), we ignore a speculator’s cost of reversing a position when he liquidates and exits the market. This assumption simplifies the exposition greatly, but all our results continue to hold qualitatively even if we work with a more complicated model for liquidation.

For a speculator maximizing profits from a short-horizon strategy, the objective function is

$$\bar{x}_k = \arg \max \ E ( \bar{x}_k' (P_{k,2} - P_{k,1}) | F_k) ,$$

$k = \{e, 1, ..., N\}$, where $F_k$ represents the information set of the speculator, i.e. the relevant idiosyncratic factor signal(s) and the systematic factor signal. Recall that all speculators obtain identical systematic factor signals, while speculator idiosyncratic factor signal differs from market to market. Since any short-horizon strategy relies on the speculator foreseeing price changes in other markets (and thus anticipating information flow into his own market when the market-maker revises price, based on those price changes), they necessarily involve an over-weighting in the systematic factor. In particular, we focus on short-horizon strategies that involve zero weight on the idiosyncratic factor and identical weights on the systematic factor, for all speculators.

Recall that $q_k$ denotes the total order flow in market $k$ on date 1, and is the sum of the speculator order flow, $x_k$, and liquidity order flow, $z_k$. Therefore if the conjectured demand of a speculator in underlying market $i$ is $x_i = \bar{\theta} (\gamma)$, the price set by the underlying market maker on date 1 is

$$P_{i,1} = \frac{cov (\epsilon_i + b_i \gamma, \bar{\theta} \gamma + z_i)}{var (\bar{\theta} \gamma + z_i)} q_i = \frac{b_i \bar{\theta} var (\gamma)}{\bar{\theta}^2 var (\gamma) + var (z)} q_i .$$

Similarly, if the conjectured demand for the speculator in the ETF market is $\bar{\theta} \gamma$, the price
set by the ETF market maker on date 1 is

\[ P_{e,1} = \frac{\text{cov} \left( \sum_{j=1}^{N} w_j (\epsilon_j + b_j \gamma), \bar{\theta} \gamma + z_e \right)}{\text{var} \left( \bar{\theta} \gamma + z_e \right)} q_e = \frac{\sum_{j=1}^{N} b_j w_j \bar{\theta} \text{var} (\gamma)}{\bar{\theta}^2 \text{var} (\gamma) + \text{var} (z)} q_e. \] (17)

On date 2, market markers across all markets have the same information set, since every one of them observes prices in all the markets. Let \( g(\gamma) \) denote the density of the systematic factor random variable, and let \( f_k(q_k|\gamma) \) represent the conditional density of the order flow \( q_k \) given \( \gamma \). Then market makers’ posterior density for the systematic factor, after observing all the prices, is

\[ g(\gamma|P_{e,1}, P_{i,1}, \ldots, P_{i,N}) = \frac{g(\gamma) f_e(q_e|\gamma) f_1(q_1|\gamma) \ldots f_N(q_N|\gamma)}{\int f_1(q_1|\gamma) \ldots f_N(q_N|\gamma) f_e(q_e|\gamma) g(\gamma) d\gamma}. \] (18)

Since all variables are normally distributed, standard methods can be used to obtain the posterior distribution analytically. This gives us the date 2 prices in the underlying markets,

\[ P_{i,2} = b_i E[\gamma|P_{e,1}, P_{i,1}, \ldots, P_{i,N}] = b_i \frac{\bar{\theta} \text{var} (\gamma)}{\text{var} (z) + (N + 1) \bar{\theta}^2 \text{var} (\gamma)} \left( q_e + \sum_{j=1}^{N} q_k \right), \forall i \in \{1, \ldots, N\}, \] (19)

and the ETF market,

\[ P_{e,2} = \sum_{j=1}^{N} w_j b_j E[\gamma|P_{e,1}, P_{i,1}, \ldots, P_{i,N}] = \sum_{j=1}^{N} w_j b_j \frac{\bar{\theta} \text{var} (\gamma)}{\text{var} (z) + (N + 1) \bar{\theta}^2 \text{var} (\gamma)} \left( q_e + \sum_{j=1}^{N} q_k \right). \] (20)

The prices set by market makers on date 2 reflect the information they glean from price changes in other markets—and speculators in all markets choose their order size to maximize the expected price bump in their respective markets, from date 1 to date 2. A small order size limits the price impact on date 1. However, such a choice by all speculators limits the quantum of price change in all markets, limiting speculator
profits. A large order size, on the other hand, caps speculator profits because of the large price impact on date 1. In effect, the choice of order size for short-horizon strategies is a co-ordination game among speculators where they need to balance these contrasting imperatives, and the optimal size solves equation (15) for each speculator.

For the optimal short horizon strategy to be an equilibrium strategy for a speculator, it must be more profitable than the “long-horizon” strategy of holding the asset till liquidation value. When ETF speculators hold assets for the long-horizon, they maximize the objective functions in equations (3) and (4). We have already solved the ETF speculator’s long-horizon problem in Proposition 1 of the previous section. The optimal long-horizon strategy of speculators in underlying markets can be obtained similarly, to give,

\[ \nu_i = \frac{1}{2\lambda_{i,1}} \text{, } \theta_i = \frac{b_i}{2\lambda_{i,1}} \text{ and } \lambda_{i,1} = \frac{1}{2} \sqrt{\frac{\text{var}(\epsilon_i) + b_i^2 \text{var}(\gamma)}{\text{var}(z_i)}}. \]  

Speculators compare expected profits from short and long-horizon strategy, and if they expect profits from the short-horizon strategy to be higher, they liquidate their position on the intermediate date itself, exiting the market before final asset values are realized. Since a short-horizon equilibrium involves all speculators trading on the same signal (the systematic factor signal), this is a classic case of rational herding. As Froot, Sharfstein and Stein (1992) describe, though rational, such herding equilibrium are usually welfare inefficient because asset prices do not reflect fundamentals. In our context, if speculators close their positions at the intermediate stage, the weights they choose are decoupled from fundamental asset value. This dislocation of prices can have significant negative real effects because asset prices are a key factor in capital allocation decisions and managerial decision making.

**Proposition 4. (Herding equilibrium)** If all speculators use short-horizon strategies, there exists an equilibrium where speculators use only the systematic factor signal to
determine their order size. The equilibrium order size for all speculators is \( \bar{\theta}, \gamma \), with \( \bar{\theta} = \sqrt{\text{var}(z) / \text{var}(\gamma)} \), and the equilibrium market maker prices are given by the equations (16), (17), (19) and (20).

For speculators, the expected profit from this short-horizon strategy is higher than the long-horizon strategy of holding the asset till liquidation value when the idiosyncratic and systematic factor signals, \( \epsilon_i \) and \( \gamma \), satisfy the following conditions:

(i) \( \left( \frac{\epsilon_i + b_i \gamma}{\gamma} \right)^2 \leq \frac{N b_i}{N + 2} \sqrt{\frac{\text{var}(\epsilon_i) + b_i^2 \text{var}(\gamma)}{\text{var}(\gamma)}} \quad \forall i = \{1, \ldots, N\} \),

(ii) \( \left( \frac{\sum_{i=1}^N w_i \epsilon_i + \sum_{i=1}^N w_i b_i \gamma}{\gamma} \right)^2 \leq \frac{N}{N + 2} \left( \sum_{i=1}^N w_i b_i \right) \sqrt{\frac{\sum_{i=1}^N w_i^2 \text{var}(\epsilon_i) + \left( \sum_{i=1}^N w_i b_i \right)^2 \text{var}(\gamma)}{\text{var}(\gamma)}} \).

Condition (i) and (ii) in the proposition above check that the expected profit from short horizon trading is at least as high as the profit expected from holding the asset till liquidation for speculators in the ETF market and all underlying markets. As the following example demonstrates, the conditions imply that herding occurs in a limited but non-empty set of scenarios.

Example. Consider an ETF with two equally weighted underlying assets, \( A \) and \( B \) (i.e. \( w_A = w_B = 0.5 \)). Let the idiosyncratic factors \( (\epsilon_A, \epsilon_B) \) and systematic factor \( (\gamma) \) be random realizations from independent standard normal distributions \( \mathcal{N}(0, 1) \). Finally, let the asset betas be \( b_A = 1 \) and \( b_B = 2 \), and the demand from noise traders \( z \sim \mathcal{N}(0, 1) \). Given these values, we are in a position to calculate, numerically, values for all the variables that guide the decisions of market participants in our model.

First off, Proposition 1 and equation (21) give the optimal long-horizon weights and
prices in the markets,

Market A:  
\[ P_{A,1}^{\text{long}} = 0.71 \cdot q_A, \quad x_A = 0.71 \cdot \epsilon_A + 0.71 \cdot \gamma, \]

Market B:  
\[ P_{B,1}^{\text{long}} = 1.12 \cdot q_B, \quad x_B = 0.45 \cdot \epsilon_B + 0.89 \cdot \gamma, \]

ETF Market:  
\[ P_{e,1}^{\text{long}} = 0.83 \cdot q_e, \quad x_e = 0.3 \cdot \epsilon_A + 0.3 \cdot \epsilon_B + 0.9 \cdot \gamma. \]

Similarly, Proposition 4 and equations (16), (17), (19) and (20) give the optimal short-horizon (herding) weights and prices in the markets,

Market A:  
\[ P_{A,1}^{\text{short}} = 0.5 \cdot q_A, \quad P_{A,2}^{\text{short}} = 0.25 \cdot (q_A + q_B + q_e), \quad x_A = \gamma, \]

Market B:  
\[ P_{B,1}^{\text{short}} = q_B, \quad P_{B,2}^{\text{short}} = 0.5 \cdot (q_A + q_B + q_e), \quad x_B = \gamma, \]

ETF Market:  
\[ P_{e,1}^{\text{short}} = 0.75 \cdot q_e, \quad P_{e,2}^{\text{short}} = 0.38 \cdot (q_A + q_B + q_e), \quad x_e = \gamma. \]

On receiving the idiosyncratic and systematic factor signals, speculators can calculate expected profit from long-horizon and herding strategies using equations (23), (24), (25) and (26),

Market A:  
\[ EP_{A,\text{long}} = 0.35 \cdot (\epsilon_A + \gamma)^2, \quad EP_{A,\text{short}} = 0.25 \cdot \gamma^2, \]

Market B:  
\[ EP_{B,\text{long}} = 0.22 \cdot (\epsilon_B + 2\gamma)^2, \quad EP_{B,\text{short}} = 0.5 \cdot \gamma^2, \]

ETF Market:  
\[ EP_{e,\text{long}} = 0.075 \cdot (\epsilon_A + \epsilon_B + 3\gamma)^2, \quad EP_{e,\text{short}} = 0.38 \cdot \gamma^2. \]

Herding occurs when speculators in all markets find short horizon strategies more profitable than long horizon strategies. This is encapsulated in conditions (i) and (ii) in Proposition 4, and in this example, they imply that the idiosyncratic and systematic
factor signals must satisfy the following three constraints simultaneously\textsuperscript{15},

\[
\frac{(\epsilon_A + \gamma)^2}{\gamma^2} \leq 0.71, \quad \frac{(\epsilon_B + 2\gamma)^2}{\gamma^2} \leq 2.24, \quad \frac{(\epsilon_A + \epsilon_B + 3\gamma)^2}{\gamma^2} \leq 4.97.
\]

Figure 5 shows the region in the factor realized-value space where these constraints are satisfied. When $\epsilon_A$, $\epsilon_B$ and $\gamma$ realizations lie simultaneously in this region, a short-horizon strategy is more profitable than long-horizon strategy for all speculators. In that case, speculators implicitly end up co-ordinating—using only systematic factor signal for calculating the order size ($x_A = x_B = x, \gamma = \gamma$)— resulting in a herding equilibrium.

The specific form of the herding probability function is driven by the assumptions imposed on the probability distribution of the systematic factor and idiosyncratic factors.

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\textsuperscript{15}In this example, we round-off to two digits after the decimal when representing the outputs of calculations. However, if the output of one stage is the input to another stage, we use the exact numerical value of the output.
in our model. The important takeaway, however, is that the probability of rational herding—while non-zero—is not 1. This is the reason herding is not an “everyday occurrence” in the ETF universe. It happens only when the parameter values line up in particular ways. Another interesting consequence follows from the observation that the conditions in Proposition 4 need to be satisfied by every single asset in the ETF. Thus the probability of herding is decreasing in $N$, the total number of assets in the ETF. This is in contrast to the phenomenon of market instability, which increases as one increases the number of assets in the ETF.\textsuperscript{16} Finally, observe that—given the distributional assumptions imposed on asset values—the probability of the herding equilibrium described in Proposition 4 is “well-defined” only for an ETF with positive beta assets (the left hand side of condition (i) in Proposition 4 is always positive, while the right hand side becomes negative when $b_i$ is negative). For ETFs with negative beta assets, a herding equilibrium is likely to take on a more complicated form and we hope to explore the details in future work.

Observe that in the herding equilibrium of proposition 4, asset prices do not contain any idiosyncratic fundamental information, so in a strict sense, this equilibrium does not possess the propagation-of-unrelated-idiosyncratic-shock feature.\textsuperscript{17} Yet, in a certain sense, the overall outcome of herding is the same as propagation of unrelated shocks: asset markets are more coupled and asset price movements are not related to change in fundamentals.\textsuperscript{18}

The dynamics of price formation in the herding equilibrium is quite different from the

\textsuperscript{16}Though we do not model the probability of market instability explicitly in the previous section, it is easy to see that it increases in $N$. Higher the number of assets in the ETF, higher the likelihood that an idiosyncratic shock affects the ETF price, which is then transmitted to other assets in the ETF through the mechanism described in the previous section.

\textsuperscript{17}Propagation of unrelated shocks is a feature of the equilibrium when speculators use long-horizon strategies. Herding is a feature of an equilibrium with short-horizon speculator strategies.

\textsuperscript{18}As is true for many models based on the Kyle (1985) framework, our results in this section come with a caveat. The herding results are obtained in a so-called “one-period Kyle” framework. In other words, we compare informed trader profits from a one-period short-horizon herding strategy to a one-period long-horizon strategy of holding the asset position till liquidation date. We believe that our results must extend to multi-period settings, but we don’t yet have a rigorous proof for this.
standard Kyle (1985) type equilibrium where speculators hold the asset till liquidation value is realized. In a certain sense, speculators in a herding equilibrium behave like participants in a Keynesian beauty contest. In our setup, short-horizon strategies are profitable because of the inter-market learning among ETF and underlying markets. When underlying market $i$’s speculator puts higher relative weight on the systematic factor, it gets to have a stronger effect on intermediate ETF price, $P_{e,2}$ (because the ETF market maker learns from underlying markets before setting $P_{e,2}$). This leads to ETF speculators putting more weight on the systematic factor. Since underlying market makers learn from the ETF price, in turn this pushes a speculator in market $j$ to put higher weight on the systematic factor. A full blown feedback cycle can ensue, leading to an equilibrium with no weight on the idiosyncratic factor, for all speculators. Thus rational herding in the ETF ecosystem may provide a partial explanation for sudden changes in systematic factor risk premium in response to market conditions.

6 A Simple Case Study

As a simple case study of the linkages discussed above, we relate our analysis to the intriguing behavior of ETFs during the Greek debt crisis in summer 2015. We caution that this example is best viewed not as a rigorous empirical validation, but more along the lines of a heuristic exposition to illustrate features of our model. As a useful preliminary, Figure 6 depicts the movements in the main Greek stock index for the period 2014-15.

The Greek ETF on NYSE (Ticker Symbol: GREK) tracks the FTSE/ATHEX Custom Capped Index, which is a market capitalization weighted index of the 20 largest companies on the Athens stock exchange (Athex).\textsuperscript{19} It is one of only two major global ETFs that provide international investors with exposure to Greece, the other being the Lyxor ETF

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Figure 6: Greek Stock Index, 2014-15

FTSE Athex 20, traded in Europe. At the height of the Europe-Greece bailout crisis in summer 2015, the Greek stock markets and the European Lyxor ETF shut down from June 29 to August 02, 2015. During this period, the Greek ETF on NYSE, as well as foreign listings of some prominent Greek companies continued to trade, but there were no new creations or redemptions of ETF units. In terms of our model, during this phase, the foreign venues of the Greek companies (foreign listings on London Stock Exchange (LSE) and American Depository Receipts (ADRs)) constituted the underlying markets, while the NYSE platform trading GREK was the ETF market. Since underlying stock markets were closed, the Greek ETF on NYSE was an important venue for price discovery of Greek stocks. Thus the conditions seemed propitious for the kind of phenomena

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Figure 7: Greek ETF on NYSE and its major constituents when Greek markets were closed during June 29–August 02, 2015

Though GREK tracks the value weighted sum of the top 20 companies on the Athens exchange, the top three constituents — Coca Cola Hellenic Bottling Company (HBC), Hellenic Telecom, and National Bank of Greece — comprised close to 45% of the ETF’s holding on 29 June, 2015, when the exchange closed down.\(^{21}\) Coca Cola HBC and National Bank of Greece continued to trade throughout the period on their alternative foreign venues — LSE and NYSE ADR respectively — notwithstanding the Athens exchange shutdown. However, trade in Hellenic Telecom’s alternative venue, the US pink sheets,

\(^{21}\)The precise holdings were: Coca Cola HBC 25.05%, Hellenic Telekom 9.94%, and National Bank of Greece 9.36%. Source: http://www.globalxfunds.com/GREK
Figure 8: Greek ETF on NYSE and its major constituents after Greek markets re-opened on August 03, 2015
Table 1: Correlation in the daily percentage price change between ETF: GREK and its major constituents when Greek markets were closed. (June 29–August 02, 2015)

<table>
<thead>
<tr>
<th></th>
<th>ETF: GREK</th>
<th>CocaCola HBC</th>
<th>National Bank of Greece</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETF: GREK</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CocaCola HBC</td>
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<tr>
<td>National Bank of Greece</td>
<td>0.767</td>
<td>0.558</td>
<td>1</td>
</tr>
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</table>

*Hellenic Telecom was not included in the above table because trade in its alternative venue, the US pink sheets, was suspended from June 29 till July 31, 2015.

Table 2: Correlation in the daily percentage price change between ETF: GREK and its major constituents after Greek markets re-opened (August 03–December 31, 2015)

<table>
<thead>
<tr>
<th></th>
<th>ETF: GREK</th>
<th>CocaCola HBC</th>
<th>National Bank of Greece</th>
<th>Hellenic Telecom Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETF: GREK</td>
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<tr>
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<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>National Bank of Greece</td>
<td>0.371</td>
<td>0.192</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Hellenic Telecom Company</td>
<td>0.368</td>
<td>0.451</td>
<td>0.395</td>
<td>1</td>
</tr>
</tbody>
</table>

was suspended from 29 June till 31 July. These three companies belong to different sectors, and their exposure to Greece varies widely: 95% of Coca Cola HBC’s sale was outside Greece, 56% of National Bank of Greece’s revenue was earned outside Greece, and 36% of Hellenic Telecom Company’s revenue was from sale outside Greece, in 2014.22

Thus one would not expect the effect of the Greek bailout crisis to be similar on all firms. Yet as Figure 7 shows, from June 29 to August 02, all prices seemed to move largely in-sync, suggesting propagation-of-shocks and herding. Figure 8 and Tables 1 and 2 indicate that price co-movement fell substantially after Greek markets re-opened, supporting our hypothesis.

22Source: Company Financial Reports
7 Conclusion and Policy Implications

When the S&P Depository Receipt (SPDR) — the first ETF — was launched in 1993, it was a sideshow to underlying markets. Money in ETFs came mostly from passive long term investors seeking an alternative to mutual funds. There was little independent information in those markets and underlying market players took little notice of them. There was little danger of phenomenon of the kind that we describe in this paper affecting the markets. A lot has changed over the years. While ETFs still remain small relative to the underlying in highly liquid markets, in other, less-liquid markets, ETFs dominate trading. Today we have ETFs on many assets which are inaccessible to traders otherwise.

In this paper, we showed that when the information content in ETF prices does not overlap perfectly with the information in underlying markets, markets can become more fragile. A worrisome problem for regulators is the propagation of market instability arising from the feedback effects of ETFs. In the absence of ETFs, market maker learning is more focused on own market order flow. With an ETF, however, the market maker also extracts information from the ETF price, meaning that both own market and ETF market information affects prices. This can result in greater volatility as disturbances in the ETF can affect underlying market prices, even when such information is irrelevant for a particular underlying asset. When this occurs, it is akin to the “tail wagging the dog” in that the ETF price changes the underlying prices rather than the underlying prices changing the ETF price. Moreover, we demonstrated that ETFs can introduce persistent distortions from fundamentals at the individual asset level, even while enhancing price efficiency at the aggregate ETF level. Assets with high beta and high weighting in the ETF are especially vulnerable to such distortions.

That ETFs can lead to greater market fragility is surely an issue of regulatory concern. Our model suggests some approaches to alleviate such incipient instability. One regulatory solution, for example, could be to restrict ETFs to baskets where the under-
lying assets are easily tradeable. Another could be to reduce the size of the basket, since that reduces the likelihood of noise transmission. Restricting ETF basket size, however, needs to be undertaken with caution, since smaller baskets can also make it easier for market participants to co-ordinate in certain market scenarios, increasing the likelihood of herding. Our analysis also suggests that enhancing the quality of information on underlying assets would help, since it reduces the information difference among markets. Thus, actions such as enhanced transparency of individual bond prices, perhaps through more real time trade reporting to TRACE, may be helpful. Regulators may also wish to encourage the nascent electronic trading of bonds, as greater transparency on order information as well as greater accessibility will serve to dampen ETF-induced volatility. Since information difference is a pre-condition for herding across markets, these measures are also likely to reduce the propensity of herding.
Appendix: Proof of Results

Proof of Proposition 1. Let $\lambda_e$ denote the conjectured price impact factor in the ETF market, and let $\sum_{j=1}^{N} \nu_j \epsilon_{ij} + \theta_e \gamma$ denote the conjectured size of the ETF speculator’s order, in equilibrium. Denote the ETF speculator’s order by $x'_e$. From equation (3), we obtain the ETF speculator’s maximization problem,

$$x_e = \arg \max_{x'_e} x'_e \left( \sum_{j=1}^{N} w_j (\epsilon_j + \gamma b_j) - \lambda_e x'_e \right)$$

We solve this equation to get the ETF speculator’s optimal order size,

$$x_e = \frac{\sum_{j=1}^{N} w_j \epsilon_j + \sum_{j=1}^{N} w_j b_j \gamma}{2 \lambda_e}$$

Setting this equal to $\sum_{j=1}^{N} \nu_j \epsilon_{ij} + \theta_e \gamma$ yields the equilibrium $\nu_{ej}$, $j = \{1, ... N\}$, and $\theta_e$ in equation (10).

To obtain the equilibrium price impact factor, note that

$$\lambda_e = \frac{\text{cov} \left( \sum_{j=1}^{N} \epsilon_j w_j + \gamma \sum_{j=1}^{N} w_j b_j, q_e \right)}{\text{var} (q_e)} = \frac{\sum_{j=1}^{N} \nu_j w_j \text{var} (\epsilon) + \theta_e \sum_{j=1}^{N} w_j b_j \text{var} (\gamma)}{\sum_{j=1}^{N} \nu_j^2 \text{var} (\epsilon) + \theta_e^2 \text{var} (\gamma) + \text{var} (z)}$$

Replacing the values for $\nu_e$ and $\theta_e$ from equation (10) gives the following quadratic equation in $\lambda_e$,

$$\lambda_e = \frac{2 \lambda_e \sum_{j=1}^{N} w_j^2 \text{var} (\epsilon) + 2 \lambda_e \left( \sum_{j=1}^{N} b_j w_j \right)^2 \text{var} (\gamma)}{\sum_{j=1}^{N} w_j^2 \text{var} (\epsilon) + \left( \sum_{j=1}^{N} b_j w_j \right)^2 \text{var} (\gamma) + 4 \lambda_e^2 \text{var} (z)}$$

which can be solved to obtain the equilibrium value of $\lambda_e$ in (11).

The value of $\lambda_{ei}$ is obtained from equation (7). Substituting the equilibrium values
for $\nu_{ej}$ and $\theta_{e}$, we get the following equation,

$$
\lambda_{ei} = \frac{2\lambda_e w_i \text{var}(\epsilon) + 2\lambda_e b_i \left(\sum_{j=1}^{N} b_j w_j\right) \text{var}(\gamma)}{\sum_{j=1}^{N} w_j^2 \text{var}(\epsilon) + \left(\sum_{j=1}^{N} b_j w_j\right)^2 \text{var}(\gamma) + 4\lambda_e^2 \text{var}(z)}.
$$

Substituting the previously obtained value for $\lambda_e$ in the above equation gives us $\lambda_{ei}$. ✷

**Proof of Proposition 2.** A shock of of $\eta_j$ to the idiosyncratic component of asset $j$, $\epsilon_j$, causes the ETF speculator to increase his demand by $\nu_{ej}\eta_j$. From equation (5), this implies a price jump in underlying $i$ of $\lambda_{ei}\nu_{ei}\eta_j$. Replacing the equilibrium values for $\nu_{ei}$ and $\lambda_{ei}$ from Proposition 1 gives the magnitude of the shock propagated to asset $i$. ✷

**Proof of Corollaries 1 and 2 (Proposition 2).** Follow directly from the expression for shock propagation, in equation (13). ✷

**Proof of Proposition 3.** Since all variables are normal, the posterior variance of a market maker in underlying market $i$, after seeing the ETF price change is,

$$
\text{var}(\epsilon_i + b_i \gamma|P_{ei1}) = \text{var}(\epsilon_i + b_i \gamma) - \frac{\text{cov}^2(\epsilon_i + b_i \gamma, z_e + \sum_{j=1}^{N} (\nu_{ej}\epsilon_j + \theta_{e}\gamma))}{\text{var}(z_e + \sum_{j=1}^{N} (\nu_{ej}\epsilon_j + \theta_{e}\gamma))}.
$$

(22)

Further,

$$
\text{cov} \left(\epsilon_i + b_i \gamma, w_i z_e + \sum_{j=1}^{N} (\nu_{ej}\epsilon_j + \theta_{e}\gamma)\right) = E_i \left[\left(\epsilon_i + b_i \gamma\right) \left(z_e + \sum_{j=1}^{N} (\nu_{ej}\epsilon_j + \theta_{e}\gamma)\right)\right]
$$

$$
= E_i \left[\nu_{ei}\epsilon_i^2 + \theta_{e}b_i\gamma^2\right]
$$

$$
= \nu_{ei}\text{var}(\epsilon) + \theta_{e}b_i\text{var}(\gamma),
$$

and

$$
\text{var} \left(z_e + \sum_{j=1}^{N} (\nu_{ej}\epsilon_j + \theta_{e}\gamma)\right) = \text{var}(z) + \sum_{j=1}^{N} \nu_{ej}^2 \text{var}(\epsilon) + \theta_{e}^2 \text{var}(\gamma).
$$
Substituting these values for covariance and variance into equation (22), we obtain,

\[
\text{var} (\epsilon_i + b_i \gamma | P_{e,1}) = \frac{\text{var} (\epsilon) \text{var} (\gamma)}{\text{var} (z) + \sum_{j=1}^N \nu_{ej}^2 \text{var} (\epsilon) + \theta_e^2 \text{var} (\gamma)} \left( b_i^2 \sum_{j=1}^N \nu_{ej}^2 - 2b_i \theta_e \sum_{j=1}^N \nu_{ej} + \theta_e^2 \right) + \frac{\text{var} (z) (\text{var} (\epsilon) + b_i^2 \text{var} (\gamma)) + \sum_{j=1, j \neq 1}^N \nu_{ej} \text{var}^2 (\epsilon)}{\text{var} (z) + \sum_{j=1}^N \nu_{ej}^2 \text{var} (\epsilon) + \theta_e^2 \text{var} (\gamma)} \cdot \text{var} (\bar{z}) + \sum_{j=1}^N \nu_{ej}^2 \text{var} (\epsilon) + \theta_e^2 \text{var} (\gamma) + \text{var} (\bar{z}) \left( \text{var} (\epsilon) + b_i^2 \text{var} (\gamma) \right) + \sum_{j=1}^N \nu_{ej}^2 \text{var} (\epsilon) + \theta_e^2 \text{var} (\gamma).
\]

Finally, substituting the values for \( \nu_{ej} \), \( \theta_e \) and \( \lambda_e \) from Proposition 1 into the above equation, we get expression (14).

\[\square\]

**Proof of Corollary (Proposition 3).** Follows directly from equation (22) in the proof of Proposition 3 above.

\[\square\]

**Proof of Proposition 4.** To obtain the optimal order size with short horizon strategies, we need to solve equation (15) for each speculator. Let market participants make the consistent conjecture that the optimal short-horizon order size for speculators in all markets is \( \bar{\theta} \gamma \). For the speculator in underlying market \( i \), the maximization problem is

\[
\bar{x}_i = \text{arg max} \ E \left( \bar{x}_i' (P_{i,2} - P_{i,1}) | \epsilon_i, \gamma \right).
\]

Substituting the values for \( P_{i,2} \) and \( P_{i,1} \) from equations (16) and (19), we get

\[
\bar{x}_i = N \bar{\theta} \gamma b_i \frac{\bar{\theta} \text{var} (\gamma)}{\text{var} (z) + N \bar{\theta}^2 \text{var} (\gamma)} \left/ 2 \left( \frac{b_i \bar{\theta} \text{var} (\gamma)}{\bar{\theta}^2 \text{var} (\gamma) + \text{var} (z)} - b_i \frac{\bar{\theta} \text{var} (\gamma)}{\text{var} (z) + N \bar{\theta}^2 \text{var} (\gamma)} \right) \right.\).
\]

Therefore, for the conjecture \( x_i = \bar{\theta} \gamma \) to be hold, we must have

\[
\frac{N + 2}{\text{var} (z) + (N + 1) \bar{\theta}^2 \text{var} (\gamma)} = \frac{2}{\text{var} (z) + \bar{\theta}^2 \text{var} (\gamma)}.
\]

On simplifying the above equation we get speculator \( i \)'s optimal order size, \( \bar{x}_i = \sqrt{\frac{\text{var} (z)}{\text{var} (\gamma)}} \gamma \).
Similarly, the maximization problem for the ETF speculator is

\[ \bar{x}_e = \arg \max E (\bar{x}_e' (P_{e,2} - P_{e,1}) | \epsilon_1, \ldots, \epsilon_N, \gamma) . \]

Substituting the values for \( P_{i,2} \) and \( P_{i,1} \) from equations (17) and (20), we get

\[ \bar{x}_e = N \bar{\theta} \gamma \sum_{j=1}^{N} w_j b_j \frac{\bar{\theta} \text{var} (\gamma)}{\text{var} (z) + N \bar{\theta}^2 \text{var} (\gamma)} / 2 \left( \sum_{j=1}^{N} b_j w_j \bar{\theta} \text{var} (\gamma) / \bar{\theta}^2 \text{var} (\gamma) + \text{var} (z) \right) - \sum_{j=1}^{N} w_j b_j \frac{\bar{\theta} \text{var} (\gamma)}{\text{var} (z) + N \bar{\theta}^2 \text{var} (\gamma)} . \]

Therefore, for the conjecture \( x_e = \bar{\theta} \gamma \) to be hold, we must have

\[ \frac{N + 2}{\text{var} (z) + (N + 1) \bar{\theta}^2 \text{var} (\gamma)} = \frac{2}{\text{var} (z) + \bar{\theta}^2 \text{var} (\gamma)} , \]

which gives us the ETF speculator’s optimal order size, \( \bar{x}_e = \sqrt{\text{var} (z) / \text{var} (\gamma) \gamma} \). Since \( \bar{x}_e = \bar{x}_i \forall i \in \{1, \ldots, N\} \), the initial conjecture about equilibrium short horizon order size is validated.

The equilibrium prices can be obtained by replacing \( \bar{\theta} = \sqrt{\text{var} (z) / \text{var} (\gamma)} \) in equations (16), (17), (19) and (20).

For the speculator in underlying market \( i \), the expected profit from the short horizon strategy is

\[ E (\bar{x}_i (P_{i,2} - P_{i,1}) | \epsilon_i, \gamma) = \left( \frac{b_i \bar{\theta}^3 \gamma^2 (N + 1) \text{var} (\gamma)}{\text{var} (z) + (N + 1) \bar{\theta}^2 \text{var} (\gamma)} - \frac{b_i \bar{\theta}^3 \gamma^2 \text{var} (\gamma)}{\text{var} (z) + \bar{\theta}^2 \text{var} (\gamma)} \right) . \]

Substituting \( \bar{\theta} = \sqrt{\text{var} (z) / \text{var} (\gamma)} \) in the above equation and simplifying, we get speculator’s expected profit from short horizon trading as

\[ \text{EP}_{i, \text{short}} = \frac{b_i N \gamma^2}{2 (N + 2)} \sqrt{\frac{\text{var} (z)}{\text{var} (\gamma)}} . \]
Similarly, for the ETF speculator, expected profit from the short horizon strategy is

\[ E(\bar{x}_i (P_{e,2} - P_{e,1}) | \epsilon_i, \epsilon_1, \ldots, \epsilon_N, \gamma) = \left( \sum_{j=1}^{N} b_j w_j \tilde{\theta}^2 \gamma^2 (N + 1) \frac{\text{var}(\gamma)}{\text{var}(z) + (N + 1) \theta^2 \text{var}(\gamma)} - \sum_{j=1}^{N} b_j w_j \tilde{\theta}^2 \gamma^2 \frac{\text{var}(\gamma)}{\text{var}(z) + \theta^2 \text{var}(\gamma)} \right). \]

Substituting \( \bar{\theta} = \sqrt{\frac{\text{var}(z)}{\text{var}(\gamma)}} \) in the above equation and simplifying, we get the ETF speculator’s expected profit from short horizon trading as

\[ \text{EP}_{e,\text{short}} = \sum_{j=1}^{N} b_j w_j N \gamma^2 \frac{\text{var}(z)}{2 (N + 2)} \sqrt{\frac{\text{var}(\gamma)}{\text{var}(\gamma)}}. \quad (24) \]

On the other hand, if the speculator in market \( i \) uses a long-horizon strategy, using equilibrium values for \( \nu_i, \theta_i \) and \( \lambda_{i,1} \) in equation (21), his expected profit is

\[ \text{EP}_{i,\text{long}} = (\epsilon_i + b_i \gamma - \lambda_{i,1} E[q_i | x_i]) \cdot x_i \]

\[ = (\epsilon_i + b_i \gamma - \lambda_{i,1} \nu_i \epsilon_i + \theta_i \gamma) \]

\[ = \frac{(\epsilon_i + b_i \gamma)^2}{2 \sqrt{\text{var}(\epsilon) + b_i^2 \text{var}(\gamma)}} \quad (25) \]

Similarly, if the ETF speculator uses a long-horizon strategy, using equilibrium values for \( \nu_e, \theta_e \) and \( \lambda_e \) from Proposition 1, his expected profit is

\[ \text{EP}_{e,\text{long}} = \left( \sum_{j=1}^{N} w_j \epsilon_j + \sum_{j=1}^{N} b_j w_j \gamma - \lambda_e E[q_e | x_e] \right) \cdot x_e \]

\[ = \left( \sum_{j=1}^{N} w_j \epsilon_j + \sum_{j=1}^{N} b_j w_j \gamma - \frac{\sum_{j=1}^{N} w_j \epsilon_j}{2} + \frac{\sum_{j=1}^{N} b_j w_j \gamma}{2} \right) \cdot \left( \sum_{j=1}^{N} \epsilon_{ej} + \theta_e \gamma \right) \]

\[ = \frac{(\sum_{j=1}^{N} w_j \epsilon_j + \sum_{j=1}^{N} b_j w_j \gamma)^2}{2 \sqrt{\sum_{j=1}^{N} w_j^2 \text{var}(\epsilon) + (\sum_{j=1}^{N} b_j w_j)^2 \text{var}(\gamma)}} \quad (26) \]

For the short horizon strategies to be an equilibrium, speculators must have no incentive to deviate to their long horizon strategies, i.e. \( \text{EP}_{i,\text{long}} \leq \text{EP}_{i,\text{short}} \) for all \( i \in \{1, \ldots, N\} \).
well as $\text{EP}_{e,\text{long}} \leq \text{EP}_{e,\text{short}}$. This gives conditions (i) and (ii).
References


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