The Family Ownership Decision*

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March 1, 2019

[Abstract]
Instead of holding a diversified portfolio, family owners, such as the Waltons of Wal-mart, hold large fractions of their wealth in a single stock. To explain this decision, we build a unique model of ambiguity aversion where the family's information advantage in their firm allows them to more accurately estimate value-at-risk in tail events relative to the diversified portfolio. Using an index of macroeconomic uncertainty, we find a strong, negative relation between the uncertainty beta and both family ownership and involvement. Also consistent with our predictions, we document that families with high absolute wealth or risk aversion are unlikely to exit the firm. Our analysis provides an explanation for family owners' decision to hold a concentrated stake in a single firm in countries with well-developed financial markets and legal regimes.

JEL Codes: G11; D81; L22

* We benefited from discussions with Sugato Bhattacharyya, Sudipto Dasgupta, Qiang Fu, Lars Peter Hansen, Mark Huson, Bob Kimmel, Jinfeng Lu, Chenghu Ma, Randall Morck, Shige Peng, Nagpurnanand Prabhala, Tan Wang, Bernard Yeung, Frank Yu, Luigi Zingales, and seminar participants at the National University of Singapore, the University of Alberta and Shanghai Advanced Institute of Finance (SAIF). Nan Li gratefully acknowledges the financial support of Shanghai Pujiang Talent Program (16PJC054).

Introduction

A large fraction of equity in U.S. firms continues to be held by founders and their descendants (Shleifer and Vishny, 1986). Anderson et al. (2009) report that 47% of the largest industrial firms in US retain founding family ownership with families, on average, owning almost 20 percent of their firms' outstanding equity. Conventional wisdom suggests that in developed equity markets, family shareholders should exit these firms and seek the benefits of diversification (Markowitz 1952; Burkart et al., 2003). Prior research provides plausible explanations for families holding large equity stakes, ranging from private benefits of control (Demsetz, 1983) to gains from their monitoring of the firm (DeMarzo and Urusevic, 2006). Little however, explains the widespread incidence of concentrated family ownership in countries with well-developed legal regimes and equity markets. A simple reference model shows the average founding family among Russell 3000 firms would need to capture an additional $700 million in private benefits to justify holding a single stock over the diversified portfolio. The Walton family (of Wal-Mart Stores) would need to privately capture over $2 billion per year to justify their huge ownership position (2018 proxy filing shows the family owns 50% of outstanding equity) relative to holding a well-diversified portfolio. Rational investors presumably recognize the potential for private family benefits and price this into public offerings of the firm's stock.

We propose an additional explanation – an information advantage - for undiversified founding family ownership in firms operating in developed economies with well-functioning equity markets. Intuitively, family shareholders, as insiders, have a varying information advantage over outside investors in family firms. This information advantage lowers search cost for family owners relative to outsiders and thus provides incentives for family shareholders to retain substantial, undiversified equity stakes in their firms (Van Nieuwerburgh and Veldkamp, 2010). Family owners arguably "specialize" in their firm by holding large, concentrated equity stakes[1], serving on the board of directors, and by retaining senior management positions and thereby amplify the family's information advantage and reduce the ambiguity of their investment relative to outside shareholders. Outsiders however, have difficulty in observing and directly measuring family owners' information advantage, leading them to invest in the market portfolio.

We build a unique model of ambiguity aversion that allows for variations in information advantage between family investors and outside investors, leading to several new and rich predictions on the family investment decision. Our model focuses on the idea that insiders hold an information advantage over outside investors and this advantage is more valuable in more ambiguous settings. We assume that investors possess ambiguity about both expected returns and return volatility. Intuitively, the ambiguity about return volatility captures the notion about severe losses that occur on an infrequent basis, i.e., value-at-risk in tail events (Taleb, 2007; Taylor and Williams, 2009). We then quantify the impact of ambiguity about return volatility on the investment decision of family owners. Our model shows that the information advantage possessed by the family shareholders allow them to estimate more accurately the potential loss in a bad event in the family firm relative to a diversified portfolio, so they are willing to hold a concentrated stake in a single firm. Simulation results from our portfolio choice model show that with reasonable ambiguity about return volatility, that family owners optimally invest 70 percent or more of their wealth in the family firm. The results of the simulation correspond well with findings of prior empirical research documenting that family shareholders, on average, hold about 25 percent of the firm's outstanding equity, equating to more than 69% of their wealth invested in the firm (Anderson and Reeb, 2003).

[1] Family block holders are not the only type of investor with large shareholdings in the asset markets. Robinson and Sensoy (2013) find private equity funds and venture capitalists hold less well-diversified portfolios relative to mutual funds. Choi et al. (2017) finds that concentrated investment strategies of institutional investors in international markets can be optimal and enhance risk-adjusted returns, which is consistent with the predictions of our model that investors optimally choose to hold concentrated portfolios in which they have an information advantage and amplify their advantage through learning.

Our ambiguity-based model yields several testable predictions. *First,* our model predicts that family ownership and presence persist in environments where the family has a greater information advantage or can better exploit their information advantage to help reduce the ambiguity of the investment in their firm relative to investments in other firms. In particular, in firms or industries with inverse co-movements with aggregate ambiguity, family shareholders' private information can play a more helpful role in lowering firm-specific ambiguity in their own firms relative to investments in other firms. In environments or settings with positive co-movement with aggregate ambiguity, families' private information proves less useful in reducing ambiguity.[2] Our model thus, implies that family owners concentrate their investment in a single firm in settings where their information advantage yields more benefit and/or they have a larger information advantage, for example, family owners with family-member CEOs would be less likely to exit the firm relative to family shareholders not holding the CEO position.

Second, our model predicts that family owners consider exiting the family firm or selling their equity if their perception about the ambiguity of the family firm increases relatively to outside investments. This may occur when innovation makes the current or existing technology obsolete in the sense of "creative destruction" (Schumpeter, 1942) and the family's information advantage no longer proves useful in reducing their ambiguity perception about their own firm relative to outside investments. Due to the increased ambiguity about their own firm, the family owners might envisage the worst-case expected return of their firm lower than that of the market portfolio and thus reduce their investment in the family firm or sell their shares. *Third,* counter to the classical mean-variance model, our model predicts, *ceteris paribus*, that family owners who are relatively risk averse or wealthy are less likely to exit than less risk averse or less wealthy families. Family owners would consider existing or reducing the share of wealth invested in the family firm if their perceived worst-case expected return

[2] Recent studies highlight the difficulties in empirically estimating the dynamics of return volatility and its influence on asset prices (e.g., Bollerslev et al., 2012; Drechsler, 2013). A rich literature finds that aversion about the ambiguity of expected returns helps to qualitatively explain the under-diversification puzzle and nonparticipation puzzle in stock markets (Garlappi, et al., 2007 and many others).

of their own firm is smaller than that of the outside investments. Investors with more risk aversion or wealth in general are more conservative in adjusting their investment, so the family owners more risk averse or wealthier reduce less the share of wealth invested in their own firm or less likely to exit.

To empirically investigate the predictions of our ambiguity model in explaining families' choices in holding large, concentrated equity stakes, we examine family ownership in the Russell 3000 firms (non-financial, non-utility) from 2001 through 2017. In our empirical analysis, we interchangeably use the terms ambiguity and uncertainty (Izhakian and Yermack, 2017). We estimate macro-uncertainty at the firm and industry level as the exposure to aggregate economic uncertainty of Jurado, Ludvigson, and Ng (2015, JLN hereafter).[3] They use more than 100 macro-economic indicators to develop a 1, 3, and 12-month look-ahead indices of aggregate economic uncertainty. The coefficient estimate from 60-month rolling regressions of firm (and industry) excess returns on JLN's uncertainty index is our proxy for insiders' information advantage in their own firms, i.e., the *uncertainty beta*.

What does the uncertainty beta tell us about an information advantage? The uncertainty beta is the correlation between the individual stock return and aggregate macroeconomic uncertainty and hence holds implications for insiders' information advantage over outside investors. A large, positive uncertainty beta indicates that the stock moves largely with the aggregate uncertainty of the overall economy and thus, the family's information advantage would not be beneficial in reducing the ambiguity of their own firm relative to outside firms. That is, their own firm is as uncertain or ambiguous as outside investments. A low or negative uncertainty beta indicates that the stock does not co-move with aggregate uncertainty and thus the firm retains more "firm-specific" uncertainty. In such settings, insiders' information advantage becomes more helpful in understanding firm specific uncertainty relative to outside investments. That is, family owners have less uncertainty about their firm versus the outside portfolio. In sum, high uncertainty betas indicate low or no information

[3] Bali et al. (2017) use this measure of uncertainty and find that stocks with more exposure to aggregate economic uncertainty (i.e., uncertainty beta) exhibit lower rather than higher returns. Risk averse investors demand extra compensation to hold stocks with negative uncertainty betas, suggesting that uncertainty beta is not a measure of risk exposure.

advantage and thus, family shareholders choose to invest less in their firm and are also more likely to exit relative to low uncertainty beta environments.

The empirical analysis indicates a strong match between the predictions of our model and patterns of family ownership in U.S. publicly-traded firms. In particular, we observe a strong, negative relation between the uncertainty beta and family ownership and/or presence. A one-standard deviation increase in industry uncertainty beta correlates with a 4.5% decrease in the odds of family investors maintaining a high ownership stake relative to a smaller stake. The analysis indicates that family owners are substantially more likely to retain to large, ownership stakes and presence in environments where their information advantage proves useful in mitigating uncertainty and ambiguity about their firm relative to outside firms.

We further test the model by examining 164 family exit decisions relative to a coarsened exact matched (CEM) sample of families choosing not to exit their ownership stakes to assess whether ambiguity surrounding the firm's information environment affects the choice to exit. Family members can maintain an information advantage and reduce ambiguity relative to other investors by holding senior management positions. If so, our model predicts, *ceteris paribus*, that family shareholders will be significantly less likely relative to other investors to exit the firm when holding management positions. Using Cox multivariate survival analysis, the results indicate that family owners with family-member CEOs are significantly less likely to exit the firm relative to family shareholders not holding the CEO position. Our analysis supports the model's prediction and suggests that family owners are more likely to retain their ownership stakes when they can exploit their information advantage to reduce ambiguity about the firm's prospects.

Private benefit models of insider ownership without uncertainty or ambiguity imply that family shareholders are more likely to exit when under-diversification costs are higher, i.e., families are more risk averse and wealthier. As discussed earlier, our model predicts the opposite. Strikingly, consistent with our model's predictions, we find that families are indeed less likely to exit their ownership stakes as family absolute wealth increases and family risk-aversion increases. The results are statistically and economically significant. We instrument for family risk aversion using representation by female family members on the board of directors and/or as large shareholders.[4] Our analysis suggests that families are significantly less likely to exit the firm when more female family members remain as board members and/or large shareholders.

We make three important contributions to the literature. First, our analysis contributes to the growing family firm literature by providing an explanation for family owners' decision to maintain ownership and control in countries with well-developed financial markets and strong legal regimes. Prior literature posits that family ownership arises as a response to markets or legal systems offering weak protection of shareholder rights, suggesting that family shareholders must make a trade-off between their monitoring of the firm (or signaling, commitment, asset protection, etc.) and diversifying their wealth across a broad basket of assets (Leland and Pyle, 1977; Shliefer and Vishny, 1986; DeMarzo and Urusevic, 2006). Markets in the U.S. however, offer strong shareholder protections, indicating that family owners need not make the trade-off between concentrated equity stakes and holding a diversified portfolio (Burkart et al., 2003). Yet, we observe that family shareholders continue to hold concentrated stakes in nearly one-half of U.S. firms. Our analysis indicates that founding family control in economies with well-developed equity markets need not rely on expropriation arguments but instead can represent the benefit of exploiting information advantages to reduce uncertainty. Furthermore, our model predicts that family firms are more likely to persist in industries where the information advantage is more useful to reduce uncertainty about the family firm relative to other firms, and the family firms are more likely persist if their information advantage is easier to pass on from one generation to the next, or can be amplified through management experience, all of which are consistent with prior empirical evidence in literature on family firms in the U.S.

Second, the study joins the growing literature on the important effect of ambiguity on economic decisions. By incorporating ambiguity about return volatility as well as the information advantage of

[4] Croson and Gneezy (2009) and Bertrand (2011) provide comprehensive surveys on the literature on differences in risk aversion between women and men. Borghans et al. (2009) provide experimental evidence of greater risk aversion in women relative to men, while any ambiguity aversion differences appear relatively inconsequential.

family owners, we generate substantial insights on decision makers' use of specialized knowledge or information to reduce the uncertainty of their investment. Our analysis indicates that family owners exploit their information advantage by removing model or parameter uncertainty and thus, can warrant holding a large, undiversified stake in a single firm. Barillas, Hansen and Sargent (2009) estimate the benefits of removing aggregate uncertainty regarding aggregate consumption, while we provide evidences of the benefits of reducing relative uncertainty of individual firms based on firm-level real investment decision.

Third, we present theoretical explanations and empirical evidence based on family ownership and family exit decisions that ambiguity about volatility plays an important role in investment decisions. Using only ambiguity about return volatility, we can explain the large, concentrated stakes of family investors in an economy with strong shareholder protections and low diversification costs. Our model with ambiguity about return volatility uniquely predicts that family shareholders are more likely to exit with less wealth, less risk aversion, and less experience or time-in-the-firm. Our model offers starkly different predictions from standard investment models without ambiguity or models incorporating only ambiguity about expected returns.

2. The model

Our model is motivated by the quantitative aspect of the puzzle as to why family owners invest a large proportion of their wealth in a single firm, and what drives their decision to exit from the family firm and invest in a diversified portfolio. We seek to answer these questions in a model of investment decision with ambiguity-averse investors (family owners) who choose between investing in two risky assets with different degrees of ambiguity about expected return and return volatility, namely, a single stock (family firm) and a diversified portfolio of other equities.

In the classic portfolio choice model proposed by Markowitz (1952) and Merton (1971), investors are assumed to make decisions with the knowledge about the unique probability distribution of stock prices and firm fundamentals. However, investors do not observe the true distribution or model of asset prices. They usually form multiple perceptions about the distribution or model with the information at hand and try to make optimal decisions given the existence of multiple scenarios. The quality and quantity of available information about the firm differs for different classes of investors. Inside investors of a firm, such as CEOs or family owners, have access to more accurate information about company fundamentals than outside investors and thus can make better inferences about the true distribution of asset prices. That is, inside investors have less ambiguity about the distribution of future stock prices than do outside investors. On the other hand, corporate insiders (CEOs and family owners) do not possess an information advantage with outside firms or a diversified portfolio of outside firms and thus have the same degree of ambiguity as other outside investors with respect to these investment choices.

Our model resembles the models that relate ambiguity aversion to the portfolio choice puzzles such as the nonparticipation puzzle and under diversification puzzle.[5] However, the important feature that differentiates our model from most of the existing portfolio choice models with ambiguity is that we allow ambiguity about both expected return and return volatility.[6] In the general framework with ambiguity, investors are uncertain about the entire distribution of the asset return. More importantly, decision makers with ambiguity aversion care about the ambiguity regarding the potential downfall in returns, which is captured by the ambiguity about the volatility rather than the ambiguity about the expected return. Notably, we find that ambiguity about expected return is not enough to rationalize the concentrated ownership of family owners; ambiguity regarding the volatility of the return appears to play a very important role in understanding the concentrated ownership decision. In Table A.1, we report the calibration of the share of wealth invested in the family firm and the diversified portfolio with different degrees of ambiguity about expected return and return volatility. If there is no ambiguity

[5] Our model is similar to that of Boyle et al (2012) with one critical difference, that is, in our model, the investor (family owner) are ambiguous about both expected return and return volatility, while in their model, the investors are only concerned with ambiguous expected return and know the exactly structure of covariance matrix of stock return.

[6] Easley and O'Hara (2009) also allow for ambiguity about both mean and standard deviation of returns when studying nonparticipation in stock markets.

about volatility, then even if the ambiguity about the expected return of the diversified portfolio is so high that the family owners think the minimum expected return of a single stock is 150% of that of the diversified portfolio, they will only invest 6% of their wealth in the single stock. On the other hand, if the family owners care about ambiguity surrounding volatility, then with moderate ambiguity about the volatility such that the family owners think the maximum volatility of a single firm is same as that of the diversified portfolio, then they will invest 50% of their wealth in the single firm. The impact on ownership level with respect to the relative ambiguity about volatility is much larger than the relative ambiguity about the expected return.

Our model builds on the MEU framework of Gilboa and Schmeidler (1989), and does not separate ambiguity and ambiguity aversion. Klibanoff et al. (2005) show that the maxmin-preference model is a limiting case of the smooth recursive preferences when the degree of ambiguity aversion trends to infinity.[7] This framework is useful in studying the economic equilibrium impact of changing ambiguity aversion while holding ambiguity constant or vice versa. However, due to data limitations, neither ambiguity aversion nor ambiguity is easily observed or measured, and variations in ambiguity aversion and ambiguity are often observationally equivalent. Hence, we adopt the MEU framework, which allow us to solve the model explicitly and derive rich testable implications that relate the family ownership and family exit decision to the observable characteristics of the family owner and the family firm. Furthermore, our model implications are consistent with the numerical simulation results of Klibanoff et al. (2005).

To focus on the decision of family owners, we assume the choice set of family owners contains two risky assets with different degrees of ambiguity and no risk-free asset. From a practical perspective, investors rarely find a risk free asset. Table A.2 shows for example, the average real return and volatility (of real returns) for treasury bonds versus the real return on the CRSP market index from 1995 to

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[7] Both approaches have widely varying implications for optimal portfolio allocation, Abdellaoui et al. (2011), Bossaerts et al. (2010), Hayashi and Wada (2010) and Dimmock et al. (2016) examine the effect of uncertainty on portfolio choice via experiments. Peijnenburg (2014) shows that the maxmin approach better matches the data. Ahn et al. (2014) explicitly compare the maxmin preferences and smooth preference via a portfolio choice experiment to explore which describes actual behavior and find evidence in favor of a kinked specification.

2017. The volatility of the 5-year treasury bonds is 5.93%, significantly different from zero, and the volatility of 30-year bonds is 17.61%, close to that of the CRSP stock market index (20.19%). Furthermore, we obtain similar implications on the relationship between the family ownership and characteristics of the family owner and the family firm in a model with two risky assets and a government bond.[8]

2.1 The portfolio choice with ambiguity about return volatility and expected return

We consider a static discrete-time economy with two risky assets.[9] Asset 1 refers to the stock of a single firm (family firm) and asset 2 is a diversified portfolio. The return on each risky asset is

$$r_i = e_i + \varepsilon_i, \;\; for\; i = 1, 2 \tag{1}$$

where e_i stems from the earnings and other learnable fundamentals of firm operations by those with inside information and ε_i stems from external forces that no one can understand or learn. We assume that the learnable fundamentals e_1 and e_2, and the noises ε_1 and ε_2 are independent of each other and all follow normal distributions so that the returns on both risky assets, r_1 and r_2, also follow normal distributions and are independent of each other.[10] The true values of the expected return and return volatility on both assets are denoted as $(\bar{r}_{i,0}, \sigma_{i,0})$, for i =1, 2.

In financial markets, investors do not observe the true value of expected return and volatility and rely on other data to make inferences about these values. In a general model with ambiguity, investors are uncertain about the distribution of the asset returns. With the normality assumption of the distribution, it is sufficient to assume that investors are uncertain about the mean and standard deviation of returns. We assume investors believe that the expected return of each asset i draws from

[8] See the Internet Appendix for details.

[9] Our static model abstracts from the complicated dynamics of stochastic volatility, while maintaining the assumption that investors are ambiguous about volatility if volatility is time varying. Our simple model allows us to derive testable implications from the explicit solution of the model and confront them with cross-sectional data. Peng (2006) formulates the stochastic calculus of Itô's type for Brownian motions with ambiguous volatility (G-Brownian motion). Epstein and Ji (2013) formulate a continuous-time asset pricing model with ambiguity about both volatility and drift.

[10] We maintain the normality assumption so that means and variances are sufficient statistics of the distributions. The assumption of independence is not essential to our results, with the main results continuing to hold if we allow for a correlation between returns on the family firm and the diversified portfolio.

a set of N possible values $R_i = \{\bar{r}_{i,n}\colon\ n = 1,2,\dots,N\}$, which contains the true value of the expected return ($\bar{r}_{i,0}$), for $i = 1,2$. That is,

$$\bar{r}_{i,min} \leq \bar{r}_{i,0} \leq \bar{r}_{i,max},$$

where $\bar{r}_{i,min} \equiv \underset{n=1,2,\dots,N}{min} \bar{r}_{i,n}$, and $\bar{r}_{i,max} \equiv \underset{n=1,2,\dots,N}{max} \bar{r}_{i,n}$, for i = 1, 2. In addition, we assume the investors believe that the return volatility on asset i come from a set of M possible values, $\Sigma_i = \{\sigma_{i,m}\colon m = 1,2,\dots,M\}$, which contains the true values of return volatility ($\sigma_{i,0}$), for $i = 1,2$. That is,

$$\sigma_{i,min} \leq \sigma_{i,0} \leq \sigma_{i,max},$$

where $\sigma_{i,min} \equiv \underset{m=1,2,\dots,M}{min} \sigma_{i,m}$, and $\sigma_{i,max} \equiv \underset{m=1,2,\dots,M}{max} \sigma_{i,m}$, for i = 1, 2.

We model ambiguity aversion in the maxmin expected utility (MEU) framework proposed by Gilboa and Schmeidler (1989). That is, investors care about the ambiguity regarding the distribution of returns, and choose the optimal portfolio to maximize their utility in the worse-case scenario. In this framework, there is no separation between ambiguity and aversion to ambiguity, and we allow the range of potential expected return and return volatility to capture both the ambiguity about the returns and the ambiguity aversion of the investors. Naturally, investors who are more ambiguity averse or face more ambiguity will exhibit concerns about wider ranges of potential expected returns and volatility.[11] Consequently, this simple framework allows us to represent family investors who face more ambiguity as those that tend to consider a wider range of possible values of expected returns.

We further assume that the family owners possess private information regarding the fundamentals of the family firm that allows them to reduce the ambiguity about the fundamentals of their own firm over time and through experience. However, like other outside investors, the family owners do not possess material private information about other firms in the diversified portfolio, as they do not manage these other firms. In our setup, the reduction in the ambiguity regarding the family firm relative to the diversified portfolio is captured by the shrinkage in the distance between the worst case

[11] In this optimal portfolio choice problem within the maxmin framework, it is the worse-case, risk-return tradeoff that measures ambiguity and ambiguity aversion.

and the true values of expected return and volatility. Hence, for the family owners who can exploit private information to reduce the ambiguity regarding the family firm, the ambiguity range of the expected return and volatility of family firm is tighter than that of the diversified portfolio, i.e. $\bar{r}_{2,max} - \bar{r}_{2,min} > \bar{r}_{1,max} - \bar{r}_{1,min}$ and $\sigma_{2,max} - \sigma_{2,min} > \sigma_{1,max} - \sigma_{1,min}$.

The portfolio choice problem of the ambiguity-averse family owners with CARA utility can be formulated as

$$\max_{\alpha} \min_{\bar{r}_{1,n_1}, \bar{r}_{2,n_2}, \sigma_{1,m_1}, \sigma_{2,m_2}} \left\{ \left[\alpha \bar{r}_{1,n_1} + (1-\alpha) \bar{r}_{2,n_2} \right] - \frac{\gamma W_t}{2} \left[\alpha^2 \sigma_{1,m_1}^2 + (1-\alpha)^2 \sigma_{2,m_2}^2 \right] \right\} \quad (2)$$

Let's first examine the minimization problem of (2). Since the objective function in (2) is monotonically decreasing in σ_1^2 and σ_2^2, the maximum possible standard deviation is always chosen, that is,

$$\sigma_1^* = \sigma_{1,max} \equiv \max_{m=1,2,..,M} \{\sigma_{1,m}\}$$

$$\sigma_2^* = \sigma_{2,max} \equiv \max_{m=1,2,..,M} \{\sigma_{2,m}\}$$

If the family is long in both the family firm and the diversified portfolio ($0 < \alpha < 1$), then the minimum possible mean returns are chosen for both assets. If the family shorts the diversified portfolio ($\alpha \geq 1$), then the maximum possible mean return of the diversified portfolio and minimum possible mean return of the family stock is chosen, and vice versa, that is,

$$(\bar{r}_1^*, \bar{r}_2^*) = \begin{cases} (\bar{r}_{1,min}, \bar{r}_{2,max}), & if\ \alpha \geq 1 \\ (\bar{r}_{1,min}, \bar{r}_{2,min}), & if\ 0 < \alpha < 1 \\ (\bar{r}_{1,max}, \bar{r}_{2,min}), & if\ \alpha \leq 0 \end{cases}$$

Given the solution to the minimization problem, the optimal share invested in the family business (α^*) can be then characterized as following,

$$\alpha^* = \begin{cases} 1 & if\ \bar{r}_{1,min} - \bar{r}_{2,min} \geq \gamma W_t \sigma_{1,max}^2 \\ \frac{\bar{r}_{1,min} - \bar{r}_{2,min}}{\gamma W_t (\sigma_{1,max}^2 + \sigma_{2,max}^2)} + \frac{\sigma_{2,max}^2}{(\sigma_{1,max}^2 + \sigma_{2,max}^2)} & if\ -\gamma W_t \sigma_{2,max}^2 < \bar{r}_{1,min} - \bar{r}_{2,min} < \gamma W_t \sigma_{1,max}^2 \\ 0 & if\ \bar{r}_{1,min} - \bar{r}_{2,min} \leq -\gamma W_t \sigma_{2,max}^2 \end{cases} \quad (3)$$

The ambiguity-averse investor makes decision based on the worse-case expected return and variance.

We focus on the scenario where the two risky assets have identical expected returns ($\bar{r}_{1,0} = \bar{r}_{2,0}$), but different return volatility, and the true value of the volatility of the diversified portfolio is less than that of a single stock ($\sigma_{1,0} > \sigma_{2,0}$). Hence, if ambiguity about the expected return of the family firm is less (more) than that of the diversified portfolio, then$\bar{r}_{1,min} > (<) \bar{r}_{2,min}$. It is straightforward that in the model without ambiguity, the investor would choose to invest more in the diversified portfolio with same expected return and lower risk; but in the model with ambiguity, it depends on the relative ambiguity of the two risky assets.

We compare the choice sets of investors with and without ambiguity in Figure 1. Note that if the family owners believe the minimum expected return of the family firm is larger than that of a diversified portfolio, that is, $\bar{r}_{1,min} > \bar{r}_{2,min}$, then the family owners would always hold a positive share of their wealth in the family firm unless the volatility or ambiguity about the volatility of the family firm gets large enough ($\sigma_{2,max} \to +\infty$), we refer to this case as the "non-exiting family". On the other hand, if $\bar{r}_{1,min} < \bar{r}_{2,min}$, then the family owners would consider exiting when their risk aversion, wealth and ambiguity about the expected return or return volatility of the family firm changes. We refer to this case as the "exiting family". Panel A of Figure 1 illustrates the scenario for non-exiting family owners ($\bar{r}_{1,min} > \bar{r}_{2,min}$). For investors without ambiguity aversion, the choice set is represented by (F_0, D_0), where F_0 represents the family firm and D_0 represents the diversified portfolio. Based on numerical calibration results, we find that the family owners would hold merely 3% of their wealth in the family firm (F_0). For a family with ambiguity aversion, but only with respect to expected return, (DB, FB) represents the choice set. In this case, the family owners would hold just 6% of their wealth in the family firm (F_B) if $\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$. For the family with ambiguity aversion with respect to expected return and return volatility, (DA, FA) represents the choice set. In this case, the family owners would hold over 80% of their wealth in the family firm (F_A) if $\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$ and $\sigma_{1,max} = 0.5\sigma_{2,max}$.

2.2. The family's exit decision

We find the family exit decision depends crucially on the ambiguity regarding the expected return and return volatility of the family firm relative to the diversified portfolio. When private information no longer permits the family owners to reduce the ambiguity about their firm relative to other firms, the family owners will exit. Intuitively, this indicates the family owners will exit when they become less certain about their firm's expected return and volatility relative to the diversified portfolio. If the family owners believe the worse-case expected return of the family firm is small enough or the worse-case volatility of the family firm return is large enough, then they are likely to exit. Note that solution (3) implies that the necessary conditions for family owners to exit are,

$$\bar{r}_{1,min} - \bar{r}_{2,min} \leq -\gamma W_t \sigma_{2,max}^2 \tag{4.a}$$

$$\text{or } \sigma_{1,max}/\sigma_{2,max} \to \infty \tag{4.b}$$

that is, either the ambiguity about the family-firm expected return increases such that the family owners think the worse-case expected return of the family firm is lower than the worse-case expected return of the diversified portfolio; or the ambiguity about the risk of the family firm increases dramatically.

Van Nieuwerburgh and Veldkamp (2010) show that information advantages can be amplified when investors can choose what to learn. When the family members serve as CEO, their information advantage about the firm is amplified through learning, hence the family holds a larger stake. The implication is that families with family members as CEO are less likely to exit the firm, all else equal. Note that we do not make any assumptions about investment horizons of either family or outside investors. Our model predicts that family owners with longer tenure in the firm will be less likely to exit the firm. In this context, our model provides important insights on family owners' horizons.[12] A simple comparative statics analysis under condition (4.a) implies that the family owners are *more* likely to exit as their risk aversion and wealth decreases. This prediction may seem surprising, as the classical

[12] Empirical literature on family firm argues that family owners have longer horizons than non-family firms. For instance, Anderson and Reeb (2003) posit that the long horizon of family owners leads them to make better investment choices. However, the mechanism for this longer horizon is typically not addressed.

model without ambiguity predicts the investor would invest more in a riskier asset (family firm) when they are more risk averse or wealthier. However, this prediction arises when unexpected shocks enlarge the ambiguity about the family firms such that the worse-case expected return of the family firm is smaller than the worse-case expected return of the diversified portfolio. In this situation, the family firm *appears* to be an asset with lower expected return and possibly lower risk; hence, the more risk averse family owners invest more, instead of less, in the family firm. The family owners may consider exiting in this situation and would be more likely to exit if they are less risk averse. This prediction sharply contradicts the implication of models based on agency costs or overconfidence that indicate less risk averse families are less likely to exit.

Panel B in Figure 1 illustrates the investment decision for the exiting family ($\bar{r}_{1,min} < \bar{r}_{2,min}$). For family owners with ambiguity aversion and are ambiguous about both expected return and risk (volatility of return), their choice set is represented by (D_E, F_E). Risk neutral family owners would choose to invest 100% of their wealth in the diversified portfolio (D_E). E represents the optimal portfolio of a risk-averse investor, which is between D_E and F_E. By comparing the optimal portfolio choice of a risk neutral and a risk averse investor, we find that the ownership of the family firm increases with risk aversion, that is, the family owners are more likely to exit when they are **less** risk averse.[13]

In summary, our model solution (3) implies the following hypotheses regarding the relation between the characteristics of family firms and family owners, and family ownership and exit decision.

> *Hypothesis 1.1: The family invests more in their own business in industries where it is easier for family owners to exploit private information to reduce ambiguity about the family firm relative to other firms.*
>
> *Hypothesis 2.1: The family is less likely to exit in industries where it is easier for family owners to exploit private information to reduce ambiguity about the family firm relative to other firms.*
>
> *Hypothesis 2.2: The family is less likely to exit when family owners are more risk averse.*

[13] The standard agency model implies a negative relation between (relative) risk aversion and ownership (Bitler et al., 2005). Faccio et al. (2011) emphasize how family owners may seek diversification through firm-level investments.

Hypothesis 2.3: The family is less likely to exit when family owners are wealthier.

Hypothesis 2.4: The family is less likely to exit when family members serve as CEO.

In our empirical tests, we use the exposure to aggregate economic uncertainty to identify those firms and industries in which it is easier for family owners to exploit private information to reduce ambiguity about their firm relative to other firms.

Notably, the comparative statics on the family owners' share invested in the family firm, as relative ambiguity increases, are unchanged for family owners exiting the firm and for family owners remaining in the firm. That is, the comparative statics are unchanged regardless of whether the family owners think the worse expected return of the family firm is bigger or smaller than that of the diversified portfolio. As long as the family owners develop more ambiguity about their own firm or the relative ambiguity of diversified portfolio decreases, the family owners tend to invest less in the (relatively riskier) family firm, or are more likely to exit. This arises because diversification benefits increase as the relative ambiguity of the diversified portfolio decreases.

2.3. Contrast with extant theory on the predictions of the family's exit decision

To explain the concentrated equity stakes of family owners, extant theories build on market frictions, private benefits of control, and/or non-pecuniary benefits of control. Leland and Pyle (1977) argue that family owners hold large stakes to signal firm quality. DeMarzo and Urusevic (2006) highlight the trade-off between agency considerations and diversification in the family exit decision. Shleifer and Vishny (1986) rely on private payoffs to justify foregoing the benefits of diversification. Morck and Yeung (2003) and Roussanov (2010) indicate that family owners could receive non-pecuniary benefits, thus explaining their decision to hold a large, concentrated stake. A common theme running through extant theories indicates that risk-averse investors receive compensation through other channels for holding a concentrated stake in a single firm. These theories indicate that the cost of concentrated ownership increases as risk aversion increases, as family wealth invested in the firm increases, and as the riskiness of the firm increases; suggesting that family owners will exit as family risk aversion, wealth, and industry risk increase. The hypotheses from our model (Hypotheses 2.1 –

2.3) stand in sharp contrast to the predictions from these existing models.

Investor overconfidence offers another explanation for the under diversification of corporate insiders. Odean (1998) shows that overconfident investors who believe they have precise knowledge of security value – but in actuality have less precision than believed – hold riskier portfolios than rationale investors with the same degree of risk aversion. Overconfident investors hold unrealistic beliefs about the precision of their estimates and about superior future returns. Unlike the overconfident investor, the ambiguity-averse investor in our model rationally improves the precision of their estimates using relevant information. Overconfidence models explain investor under diversification and thus, predict that family owners are more like to exit the firm as family risk aversion and wealth increase or when the family firm operates in a less risky industry. These predictions from overconfidence models again, stand in sharp contrast to Hypothesis 2.1-2.3 from our model.

Another generally accepted explanation on family ownership suggests that the family owners will exit the firm as soon as possible after going public to obtain the benefits of a diversified portfolio (Admati et al., 1994). Zingales (1995) models the family exit decision, suggesting that the family continuously reduces their stake in the firm until they hold only a control stake, which they then sell to outside investors at a premium. Similarly, Mello and Parsons (1998) describe family exits as sequential processes; sell small stakes to atomistic shareholders and then sell the control stake to an active investor at a premium price. In each of these settings, the benefits of diversification suggest that the longer the family remains in the firm, the more likely the family decides to sell their control stake and exit the firm. Our model predicts the opposite effect.

Ambiguity, by definition, is neither directly observed nor easily differentiated from risk in empirical analysis. The testable hypotheses obtained through our comparative statics however, allow us to test the importance of ambiguity on investment decision by developing a proxy for ambiguity and then partitioning family investors by the degree or level of ambiguity they face. Most of the hypotheses from our model stand in sharp contrast to predictions from models without ambiguity, i.e., opposing signs. In the next section, we setup our empirical analysis to test the hypotheses from our model.

3. Primary variable measurement

In this section, we test the implications of our model. The model yields several hypotheses that indicate that family owners' concern about return ambiguity and their information advantage that leads to the decision to hold large, concentrated equity stakes in a single firm. To examine the predictions and hypotheses of the model, we start with the Russell 3,000 largest industrial firms in the U.S. with a fiscal year-end in 2001. To collect the sample, we pull all firms from CompuStat for data-year 2001 with information available for total assets. We exclude public utilities (SIC codes 4812, 4813, and 4911 through 4991), financial firms (SIC codes 6020 through 6799), firms listed as master limited partnerships (21-firms), foreign firms, and firms with a share price less than $0.25. These firms are excluded to make our work comparable to previous literature on family ownership and because government regulation potentially affects firm ownership structure, corporate transparency, and performance.

We manually collect family-ownership data from corporate proxy statements and 10-k's from 2001 through 2017, including ownership level, dual-class share structures, voting power, and CEO type. Corporate histories (i.e., family lineage) are garnered from ReferenceforBusiness.com, FundingUniverse.com, and individual company websites.[14] To control for survivorship bias, we allow firms to exit and re-enter the sample. Consistent with previous research, family firms are those where the founding family continues to maintain a five percent or larger ownership stake in the firm (Shleifer and Vishny, 1986; Villalonga and Amit, 2006). To measure family presence, similar to McConnell and Servaes (1990), we use the fractional level of family ownership. Specifically, we determine the total number of shares held by the family (and their relatives), and divide by total outstanding shares – including both traded and untraded share classes (dual-class firms). Firms without family owners are referred to as diffuse shareholder firms, i.e., nonfamily firms with professional managers. After

[14] In designating family firms, we do not include shares held by charitable foundations as part of the family holdings. Foundations hold substantial equity stakes in several firms (e.g., Hershey Co, Eli Lilly & Co, Kellogg Co, Hormel Foods Corp, and etcetera – less than 20 firms) with the express intent of promoting public welfare rather than financially or economically benefiting family members.

developing our proxies for firm-level ambiguity, industry-level ambiguity and collecting control variables from CompuStat, our final sample consists of 2,154 unique firms from 2001 through 2017, yielding 22,426 firm-year observations.

3.1. Level of family ownership

In our first set of tests, we examine the relationship between the level of family ownership and firm-level and industry-level proxies of uncertainty. Studies with non-U.S. data typically use minimum thresholds for family ownership (e.g., 10%, 20%) to delineate between these controlling shareholders and diffuse shareholder firms (Claesssens et al., 2000; Dyck and Zingales, 2004). For this test, we choose an arbitrary benchmark of a 20% ownership stake to maintain firm control. Shleifer and Vishny (1986) indicate that the size of the equity stake needed for firm control depends on the degree of legal protection afforded to shareholders; suggesting that in the U.S., a 20% ownership would provide ample influence to control firm policies and direction. The mean ownership stake across our family firm sample (family firms only) is 24.75% of the firm's outstanding equity. We define high family ownership with a dummy variable that equals one when the family owns an equity stake greater than 20% of the firm's outstanding equity, and zero otherwise.[15]

3.2. Family owners exiting the firm

In additional analysis, we also compare firms where family owners exit their concentrated ownership stakes in a single firm (family exit firms) to those where family owners maintain steady ownership stakes. A family-exit firm is defined as a binary variable that equals one if family ownership drops from 5% or greater in year *t-1* to 3% or less in year *t*. We identify 169 cases in our sample period meeting this requirement. In the year prior to the reduction in ownership stakes, families held, on average, 9.994% of the firm. In the exit year, families hold just 0.564%, indicating that the family group sold 94.36% of their equity stake from *t-1* to *t*. Family-exit firms are matched to family firms using coarsened exact matching (CEM) based on total assets, debt ratio, ROA, growth opportunity, equity

[15] Different cut-off levels for high and low ownership (15% and 25%) provide similar results.

issuance, and volatility, resulting in 164 family-exit firms and 164 family firms (Iacus, King, and Porro, 2012).

3.3 Uncertainty Measures

We employ a 60-month rolling regressions (e.g. Bali, Brown, and Tang, 2017) to estimate the exposure to the macro-economic uncertainty or ambiguity of the firm-level or industry-level returns, that is the *uncertainty beta*. . In particular, our measure of uncertainty beta at the firm-level is the coefficient estimate (β_1) from the following factor model.

$$R_{it} - R_{Ft} = a + \beta_1(\textit{Macro Uncertainty } t_{-60} \textit{ to } t_{-1}) + \varepsilon_{i,t} \quad (5)$$

Where:

$R_{it} - R_{Ft}$ – *Firm's monthly stock return less the risk-free rate*
Macro Uncertainty – 12-month ahead macro-uncertainty measure fromJLN (2015).

The coefficient estimate on the uncertainty factor (β_1) captures the sensitivity of firm stock returns to macro-economic uncertainty as measured by JLN (2015). JLN calculate uncertainty from more than 100 macro-economic variables that exhibit a strong correlation with real activity. Their measure is arguably superior to stock market volatility measures as stock returns can change even with little or no change in real economic fundamentals, e.g., change in firm leverage, investor risk aversion or sentiment, etc. Figure 2 provides a plot of the JLN (2015) data with years on the *x*-axis and three-month and twelve-month ahead macro-economic uncertainty on the *y*-axis. We use the twelve-month ahead macro-uncertainty index and monthly stock returns for the primary analysis and conduct robustness testing with the three-month ahead indices[16]. We also use alternate factors models (market model and a five factor model) when estimating firm-level uncertainty beta (β_1).[17] The results with the

[16] Using three-month ahead indices, the results yield the same signs and inferences but are generally significant at the 5% to 10% level. For brevity, results are not shown in paper.

[17] In robustness testing, we also use alternative factor models. Section 4.3 outlines the results from the following two specifications when estimating firm- and industry- level uncertainty:

$$\textit{Monthly Firm(Industry) Returns} = a + \beta_1(\textit{Uncertainty}) + \beta_2(R_{Mt}\text{-}R_{Ft}) + \beta_3(\textit{SMB}) + \beta_4(\textit{HML}) + \beta_5(\textit{RMW}) + \beta_6(\textit{CMA}) + \varepsilon_{i,t}$$

$$\textit{Excess return} = a + \beta_1(\textit{uncertainty}) + \beta_2(R_{Mt}\text{-}R_{Ft}) + \varepsilon_{i,t}$$

other factor models are in Table V and VI and provide similar inferences. We use the simpler model shown in Equation (5) to control for the possibility that JLN's measure captures well-established factors (e.g., SMB, HML, etc.).

Table II, Panel A provides summary statistics for the inputs into the calculation of our uncertainty measure. Our sample period spans from January 2001 through December 2017, providing 204 monthly observations. The mean value of JLN's uncertainty index over this period is 0.925, with minimum and maximum values of 0.849 and 1.153, respectively, and a standard deviation of 0.056. Our sample comprises 2,154 unique firms with an average monthly return (over the 204 months) of 0.80%.

We also measure uncertainty at the industry level. Using Equation (5), we substitute monthly Fama-French 48-industry level returns for firm-level returns. The coefficient estimate on the macro-uncertainty index, then captures the sensitivity of each industry's stock returns relative to macro-economic uncertainty. Panel A of Table II shows the average adjusted return across the 48 Fama-French industry groups (over the 204 month) is 0.80%.

3.4. Proxies for the family characteristics

Our model predicts that family owners are less likely to exit the firm as their total wealth increases. Family absolute wealth levels are not directly observable. However, Anderson and Reeb (2003) note that family shareholders, on average, hold about 69% of their total wealth in their firm's stock; suggesting that the majority of family wealth resides in the firm's stock. We measure family wealth as the natural log of the family's fractional equity stake multiplied by year-end firm's total market value of equity. To measure market value for firms with an untraded class of stock (dual-class shares), we use the share price of the traded class as a proxy for the price of the untraded class.

The model further indicates that family shareholders are less likely to exit their ownership positions as family risk aversion increases. Borghans et al. (2009) and others based on experimental data, find that women are more risk averse than men. We instrument for risk aversion of family owners using the presence of women in family firms using a binary variable that equals one when a female member of the family serves on the firm's board of directors and/or is listed in the proxy statement as holding a 5% or larger equity stake in the firm and zero otherwise. Due to legacy issues of families commonly giving directorships to male family members or listing male family members as share owners, we potentially understate the influence of female family members on decision-making and thus, understate the level of risk aversion. If so, we potentially bias our results towards zero.

Our model also predicts that family owners are less likely to exit the firm if family members serve as CEO. We proxy for family management as a binary variable that equals one when either the founder or a founder's descendant serves as CEO of the firm and equals zero when an outside professional manager serves as CEO of the firm.

3.5. Control variables

We include several controls variables in our analysis. Firm size is measured as the natural log of total assets (Arora and Ceccagnoli, 2006). Leverage is long-term total debt divided by total assets (Elkamhi et al., 2014). Performance is earnings before interest, tax, depreciation and amortization (EBITDA) divided by total assets, measured at *t-1*. Volatility is the standard deviation of the growth of income before extraordinary times for the prior 20-quarters for each firm (Faccio et al., 2011). Growth opportunities are measured as the market value of equity divided by total assets (Bekaert et al., 2007). Equity issuance is measured as the sum of common and preferred stock annual issuances divided by total assets (Masulis and Korwar, 1986). All regressions, where noted, include industry-level dummies based on the Fama-French 48 industry grouping (less those groupings for financial and regulated public utilities). We obtain data and data definitions for the Fama-French industry groups from Kenneth French's website.[18] Table I provides a description of the variables used in our analysis.

3.6. Descriptive statistics

[18] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

3.6.1 Level of Family Ownership

Table II, Panel B presents summary statistics for the full sample. We present mean, median, minimum, maximum, and standard deviation values for the variables. Our measure of firm (industry) level uncertainty beta – using the simple model – is 0.003 (-0.061) for the full sample, ranging from a minimum of -1.879 (-0.723) to a maximum of 2.306 (1.034). Higher values of uncertainty beta indicate greater correlation of stock returns to the macro-economic uncertainty. Average firm size as measured by total assets is $4.771 billion but we note a great deal of variability with minimum and maximum values of $30.11 million and $81.98 billion respectively.

Columns 1, 2, and 3 of Table II, Panel C present data for non-family firms, family firms, and *t*-statistics for difference of mean tests between non-family and family firms. The full sample consists of 22,426 firm-year observations for 2,154 unique firms, comprising 15,003 (66.9%) observations for non-family firms and 7,423 (33.1%) observations for family firms. Similar to prior studies examining family ownership (Villalonga and Amit, 2006), we find that family firms tend to be smaller, less levered, and less risky than their nonfamily counterparts. In particular, family firms maintain $2.613 billion in total assets compared to $5.839 billion in nonfamily firms. Debt as a fraction of total assets is 17.3% in family firms and 20.3% in nonfamily firms. Volatility, measured as standard deviation of the growth of income before extraordinary items for the prior 20 quarters for each firm, is 36.15% in family firms and 69.50% in nonfamily firms. Family owners, on average, hold 24.75% of their firm's shares. Nonfamily firms exhibit an uncertainty beta of -0.008 and family firms have an uncertainty beta of 0.026, on average the stock return of family firms correlates more with that of nonfamily firms, the difference in uncertainty beta between nonfamily and family firms is significant at the 5% level.

We also segregate family firms into two groups; those with low ownership (greater than 5% but less than or equal to 20%) and high ownership (those with ownership >20%). Table II, Panel C, columns 4, 5 and 6 present firm characteristics for low- and high- ownership family firms along with t-statistics for difference of mean tests between low- and high- family ownership firms. In the high family ownership group, family shareholders hold, on average, 39.38% of the firm's outstanding equity. In the low family ownership group, the average equity stake is 11.17%. Firms with low- and high-family ownership tend to exhibit similar characteristics with the exception of return-on-assets. We find that low- and high- family ownership firms are similar in size (total assets), use about the same level of debt (leverage), have similar market valuations (Tobin's Q), and exhibit similar levels of firm-level and industry-level uncertainty betas. However, firms with high family ownership exhibit superior accounting performance relative to firms with low family ownership (ROA: 13.28% versus 9.89%).

3.6.2 Family Exit Firms

Table II, Panel D presents summary statistics of the matched family-exit firms and family firms. We observe a relatively homogenous match between the family-exit and family firms. Using difference of mean tests, we do not observe significant differences between the two sets of firms for total assets, debt ratio, ROA, growth opportunities, equity issuance, and volatility. In univariate analysis, firm-level and industry-level uncertainty betas do not differ significantly for family-exit firms and family firms. Firm-level uncertainty beta for family-exit (family) firms is 0.183 (0.088). Industry uncertainty beta for family-exit (family) firms is -0.004(-0.037). In the next section, we explore our central hypotheses in a multivariate framework.

4. Multivariate tests

4.1. Cross-sectional analysis of family ownership

Our first set of multivariate tests to examine the Hypothesis 1.1 exploiting cross-sectional differences in the level of ownership across the Russell 3000 industrial firms from 2001 through 2017. The model predicts that family shareholders maintain larger equity stakes in the firm where their private information is more useful in reducing the ambiguity about their own firm relative to other firms, that is, in the firm with lower uncertainty beta, or less correlation with aggregate uncertainty. We examine this proposition using the following specification:

$$Family\ Shareholder = a + \gamma_1(Uncertainty\ Beta) + \gamma_2 X_{i,t} + \varepsilon_{i,t}$$

We proxy for family shareholder with two measures. The first measure is the percentage of outstanding equity held by the family. The second measure is a dummy variable that designates high and low levels of family ownership. Ownership levels above 20% or more of outstanding shares are designated as high and less than 20% are designated as low family ownership. We measure the uncertainty beta at the firm- and industry- level as outlined in section 3.3.1. X represents a vector of control variables. Table I provides variable definitions.

The empirical analysis indicates a robust, negative relation between family ownership and uncertainty beta. Table III, columns 1 and 2, presents regression results with family ownership as the dependent variable and firm and industry uncertainty beta, respectively, as the primary explanatory variables. Column 1 shows that family ownership levels exhibit a significantly negative relation to firm-level uncertainty beta; suggesting that as the returns of the firm co-move more with the macro-uncertainty, family investors hold significantly smaller equity stakes. A one standard deviation increase in firm uncertainty beta indicates 0.266% decrease in family ownership.

Column 2 presents regression results with family ownership levels as the dependent variable and industry-level uncertainty beta as the explanatory variable. Again, we observe a significantly negative relation between family ownership and uncertainty beta. The coefficient estimate on industry uncertainty beta is -0.024 and is significant at the 1% level; suggesting that family shareholders tend to hold significantly smaller stakes in their firms in the industries that co-move more with the macro-uncertainty. A one standard deviation increase in industry uncertainty beta corresponds to a 0.81% decrease in family ownership. In the full sample, 66.9% of firms are designated as non-family firms, suggesting the coefficient estimates on uncertainty beta understate the economic impact of uncertainty on family ownership. We further investigate the economic importance by only examining family firms in our next set of tests.

In columns 3 and 4 of Table III, using only family firms, we examine the relation between family ownership and uncertainty beta using a logit specification. The dependent variable equals 0 for low-family ownership firms (>= 5% and <=20%) and 1 for high family ownership firms (>20%). The cross-sectional analyses with the logit regressions further support Hypothesis 1.1 of our model with ambiguous volatility. Specifically, family owners tend to maintain larger equity stakes when their information set allows them to better assess the risk and return of their investment (i.e., in environments where the return of firm stock co-move less with macro-uncertainty) A one standard deviation increase in firm uncertainty beta indicates 5.85%[19] decrease in the odds of a family investor maintaining a high ownership stake. In column 4, using industry rather than firm level uncertainty beta, we note that a one standard deviation increase in uncertainty beta is associated with a 4.50% decrease in the odds of family investors maintaining a high ownership stake relative to a smaller equity stake. The results provide support to H1.1.

4.2 Family Exit Analysis

The cross-sectional analysis indicates that family shareholders hold larger equity stakes in environments where return of firms co-move less with economic uncertainty, so that their information sets allow these investors to better assess risk and return of their own firms relative to other firms. Our model implies;

> *(1) family owners are less likely to exit in firms or industries with smaller uncertainty beta, where their private information is more helpful in assessing future returns and risk (H2.1),*
> *(2) family owners are less likely to exit, if more female family owners on the board, so that the family is more risk averse (H2.2),*
> *(3) family owners are less likely to exit, if the family has more wealth invested in the firm relative to other family owners (H2.3),*
> *(4) family owners are less likely to exit when the family members serve as CEO. (H2.4)*

[19] Panel B, Table II shows that under simple model, a one standard deviation of firm uncertainty beta is 0.665. Therefore, (0.912-1)*100%*0.665=- 5.85%. Similarly, for industry uncertainty beta, we get (0.866-1)*100%*0.336=-4.50%.

To examine these proposition, we examine family-exit firms versus family firms using a Cox survival rate specification:

$$\textit{(Family Time in Firm, Family Exit Dummy)} = \alpha + \gamma_{1\text{-}2}\textit{(Firm or Industry Level Uncertainty Beta}_{i,t}\textit{)} + \gamma_{3\text{-}7}\textit{(Family Characteristics}_{i,t}\textit{)} + \gamma X_{i,t} + \varepsilon_{i,t}$$

The variables are defined in Table I with X representing a vector of control variables. In the Cox survival model (Lane et al., 1986), we relate the time that passes before the family exits to our series of covariates on firm (industry) uncertainty beta and family characteristics that potentially affect the family's exit decision. For example, we examine whether family management influences the family's decision to exit their equity ownership stake. We present the coefficient estimates as odds ratios. An odds ratio that is greater (less) than 1.0 indicates that the family is more (less) likely to exit the firm relative to the independent variable, e.g., a coefficient estimate on family management of 0.29 indicates that family firms with a family manager or only 29% as likely to exit the firm as a family firm without a family manager. We control for serial correlation and heteroskedasticity using the Huber-White sandwich estimator (clustered on firm-level identifier) for the standard errors on the coefficient estimates. In robustness, testing, we also use exponential and Weibull hazard specifications rather than a Cox specification and find similar results and inferences.

Table IV displays the results for family-exit firms versus the matched sample of family firms. Columns 1 through 4 incorporate firm-level uncertainty beta and columns 5 through 8 use industry-level uncertainty beta. The results, across all of the specifications, indicate that family investors are more likely to exit their ownership position as uncertainty beta increases. We find that family investors are about 21.28% to 29.93% more likely to exit the firm with a one standard deviation increase in firm-level uncertainty beta.[20] Columns 5 through 8 continue to indicate that family owners are about

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[20] We calculate the economic influence of a one standard deviation in firm level uncertainty as: (coefficient estimate on firm level uncertainty – 1.0) x (1-standard deviation change in firm level uncertainty). For model 1 in Table IV, this is: Increase in the probability of a family exit = (1.45 – 1.0) x 0.665*100% =29.93%.

21.84% to 27.89% more likely to exit their firms with a one standard deviation increase in industry (rather than firm) uncertainty beta. The results on the family exit decision on firm- and industry- level uncertainty beta are significant at the 5% or better level across all of the specifications and suggest that uncertainty plays an economically important role in families' decisions to maintain their equity stakes. The exit results confirm the cross sectional analysis and provide evidence consistent with our model's prediction.

Columns 2, 3 and 4 of Table IV display the Cox regression results when examining family characteristics while controlling for the level of firm uncertainty beta. Columns 6, 7, and 8 show similar results but we control for the level of industry uncertainty beta rather than firm uncertainty beta. Because the results across firm- and industry- uncertainty provide similar and near identical inferences, we restrict our discussion to columns 2, 3, and 4 that control for firm uncertainty.

The model predicts that family owners with private information more helpful to reduce the ambiguity will be less likely to exit the firm relative to families with less helpful private information (H2.1). Family members serving as CEO of the firm arguably provides the family shareholders with more helpful private information regarding the firm's future prospects versus an external professional manager serving as CEO. If so, we expect firms with family CEOs to be less likely to exit their ownership positions. Column 2 of Table IV presents the family-exit results when a family member serves as CEO. The analysis indicates that family exits are only 29% as likely to exit the firm when a family member serves as CEO as compared to an external CEO; suggesting that families with more helpful private information tend to remain in the firm relative to families with less private information. The results provide support to H2.4.

Our model also predicts that family owners should be less likely to exit their ownership stake as family risk aversion increases, i.e., families will maintain their ownership position as their level of risk aversion increases, all else equal. We instrument for risk aversion using a dummy variable if a female member of the family serves on the board of directors and/or holds a 5% or larger stake in the firm

(Borghans et al., 2009). The analysis strongly indicates that family exits are far less likely to occur as family risk aversion increase (female representation increases). Column 4 of Table IV displays the results and suggests that more risk averse families (those with female board representation) are only about 29% as likely to exit the firm as less risk averse family investors (no female board representation).[21] Our results on family risk aversion support Hypothesis 2.2 of the model; all else equal, families with greater risk aversion tend to stay invested in the firm.

Our model predicts that wealthier families are less likely to exit the firm than less wealthy families, i.e., families with a larger absolute, dollar wealth stake should be less likely to exit than families with smaller absolute wealth stakes. The results of the analysis indicate that a one-standard deviation in the absolute value of family wealth invested in the firm yields a 25% decrease in the likelihood of a family exit. Our analysis supports Hypothesis 2.3 of the model and indicate that as family invested wealth increases, family owners are less likely to exit the firm.

Overall, the family-exit analysis based on firm- and industry- uncertainty beta and family characteristics supports the predictions of the model. Specifically, our empirical analysis indicates that family owners are less likely to exit their ownership positions in the firm or industry co-move less with macro-uncertainty (smaller firm-/industry- level uncertainty beta), where their private information is more helpful, when possess more helpful private information (family management), exhibit greater risk aversion (female board representation), and as their invested wealth in the firm increases.

4.3 Robustness Testing

4.3.1 Cross-sectional Robustness Testing

In section 4.2, we examined the relation between the levels of family ownership and firm and industry uncertainty beta. For that analysis, we used only the 12-month ahead JLN (2015) uncertainty

[21] An alternative explanation for our finding on family female representation centers on the notion that because women are more risk averse than men, family owners may be more willing to place female family members on board in sectors characterized by high (or low) risk. We examine this proposition by regressing firm volatility on female-family representation and our control variables (with and without industry controls). The analysis indicates that female-family representation is not significantly related to firm risk (volatility).

index in our computation of firm and industry uncertainty beta. Table V, panel A shows the cross sectional results when using the JLN index plus a market factor. Panel B of Table V displays cross sectional results when using the JLN index factor plus a Fama-French 5-factor model (market factor, SMB, HML, RMW, and CMA). Our main results are robust to the different ways of computing uncertainty beta, whether with the market factor or with the Fama-French 5-factor model. As the level of firm (or industry) uncertainty beta increases, family owners hold smaller equity stakes in their firms. Notably, the coefficient estimates reported in our primary analyses (Table III) and those reported in the robustness tests are of similar magnitude, same sign, and same significance, suggesting that construction of our uncertainty measure tends to relatively insensitive to the specification. Overall, our cross sectional analyses indicate that as firm and industry uncertainty betas increase, family shareholders hold smaller stakes; providing evidence consistent with the predictions of the model.

4.3.2 Family-Exit Robustness Analysis

The primary family-exit analysis in section 4.2 used only the 12-month ahead JLN, (2015) uncertainty index in the computation of firm and industry uncertainty beta. Table VI, panel A, shows the family exit results when using the 12-month ahead JLN index and to compute firm- and industry-level uncertainty beta. Panel B shows the results when using JLN index and to compute firm- an industry- level uncertainty beta.

The family-exit analysis with the alternate constructions of firm and industry uncertainty beta continues to indicate that family owners are significantly more likely to exit their ownership stakes as the level of firm and industry uncertainty beta increases. The results when including the market factor indicate that family investors are about 26.69% (21.50%) to 46.22% (29.25%) more likely to exit their stakes with a one-standard deviation in firm (industry) uncertainty beta. Consistent with the earlier analysis, we also find that family owners are substantially less likely to exit their ownership stakes when having a private information advantage (family management), with greater risk aversion (female board representation), and with greater wealth invested in the firm. Similarly, the family-exit results remain unchanged with using the Fama-French 5-factor model (rather than just the JLN index and the market factor) to compute firm- and industry- level uncertainty betas. Overall, the family-exit robustness testing indicates that family owners are more likely to exit when having more helpful private information in reducing the ambiguity of their own firm relative to other firms, either in the firm or industry with less uncertainty beta or when family CEO presents, greater levels of risk aversion, and greater levels of absolute wealth invested. The results are consistent with the predictions of the model.

5. Summary and conclusion

Classical investment theory and conventional financial advice suggests that investors should hold well-diversified portfolios instead of holding large, concentrated stakes in a single firm. Yet, we continue to observe that founding families of many publicly traded firms invest the bulk of their wealth in just one stock in well-developed equity markets such as those in the U.S. With the ability to control senior management posts, board seats, and the prevalence of super-voting dual-class shares, the notion that family shareholders need to maintain such large stakes to extract private benefits appears to be puzzling.

We propose an information advantage explanation for families holding large, concentrated stakes based on decision theory in which investors have concern over ambiguity about both the expected return and return volatility and they can exploit specialized knowledge to reduce the relative ambiguity.

Using simulation analysis, we show that for family owners, with private information about the firm to reduce the ambiguity about their own firm relative to outside firms, this can result in a portfolio where most of their investable funds are held in the family firm. In contrast, atomistic shareholders without access to private information invest in a diversified portfolio. We find that ambiguity about return volatility is critical to explain quantitative aspect of the puzzle that family shareholders maintain large stakes in the family firm.

Furthermore, our model provides rich testable implications, including several that appear contrary to standard expectations in the literature on family exiting decision. The model predicts that family owners are more likely to retain their ownership stakes where private information and experience allows the family to understand better the nature of the firm's prospects. The model distinctly predicts that families will be less likely to sell-off their ownership positions (exit the firm) as family wealth invested in the firm increases[22]; as family (absolute) risk-aversion increases and; as family tenure in the firm increases.

Finally, we empirically investigate the model's predictions using two testing procedures. First, we examine family ownership, in the cross-section, for a large sample of family held U.S. industrial firms. Second, in perhaps a more robust test environment, we examine family decisions to sell-off their ownership stakes and exit the firm. Our first series of test, the cross-sectional analysis, suggests that families hold larger ownership stakes in firms and industries with less uncertainty beta, in which the privation information is more useful in reducing the relative ambiguity of family firms. The second series of tests examining family exits also confirms the predictions of our ambiguity model. Notably, the exit analysis also indicates that families are less likely to sell-off their ownership stakes as their invested wealth increases, as their risk aversion increases, and when the family actively manages the firm. In sum, the empirical analysis supports the models' predictions, suggesting that families rationally choose to hold an undiversified portfolio when their private information allows them to reduce ambiguity about the firm's prospects relative to outside firms.

Our analysis is motivated by the quantitative feature of the family ownership puzzle; that is, why family owners defy conventional finance wisdom by maintaining large, concentrated stakes in a single firm. We show that information advantage on the family firm allows family owners to reduce the ambiguity about expected return and especially about return volatility of the family owned stock

[22] Bitler et al. (2005) argue that the standard agency model also implies a positive relationship between wealth and ownership, as absolute risk aversion is typically thought to decreasing in wealth. However, for investors with constant absolute risk aversion, this implied positive relationship between wealth and ownership does not hold.

relative to that of a diversified portfolio of other firms. Yet, family block holders are not the only type of investor with large shareholdings in the asset markets. Robinson and Sensoy (2013) find private equity funds and venture capitalists hold less well-diversified portfolios relative to mutual funds. A recent study by Choi et al. (2017) finds that concentrated investment strategies of institutional investors in international markets can be optimal and enhance risk-adjusted returns, which is consistent with the predictions of our model that investors optimally choose to hold concentrated portfolios in which they have an information advantage and can amplify their advantage through learning.

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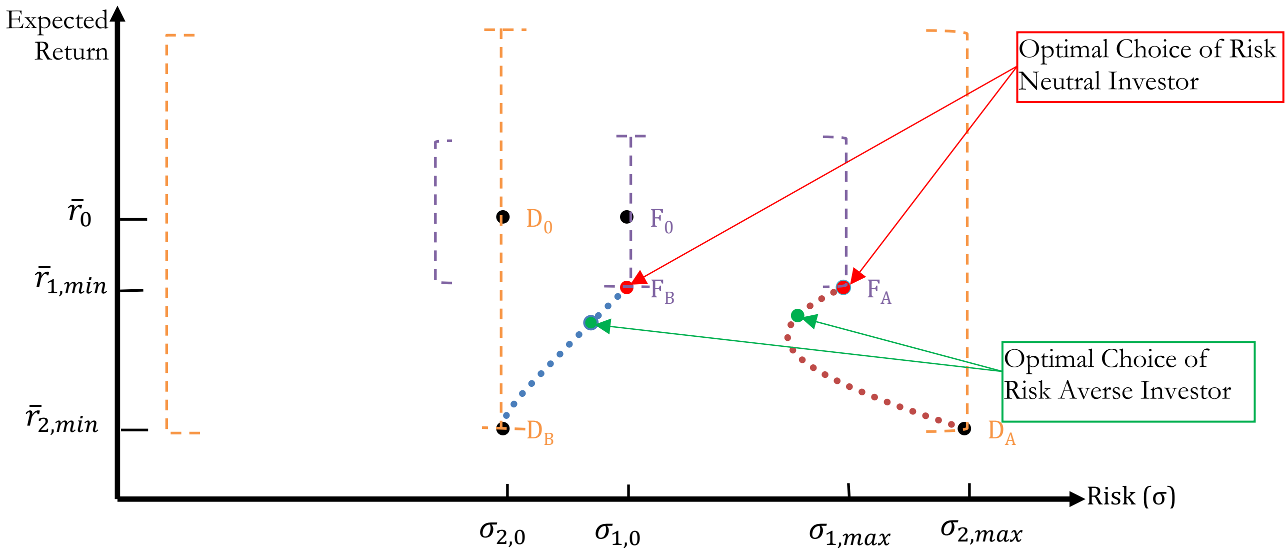
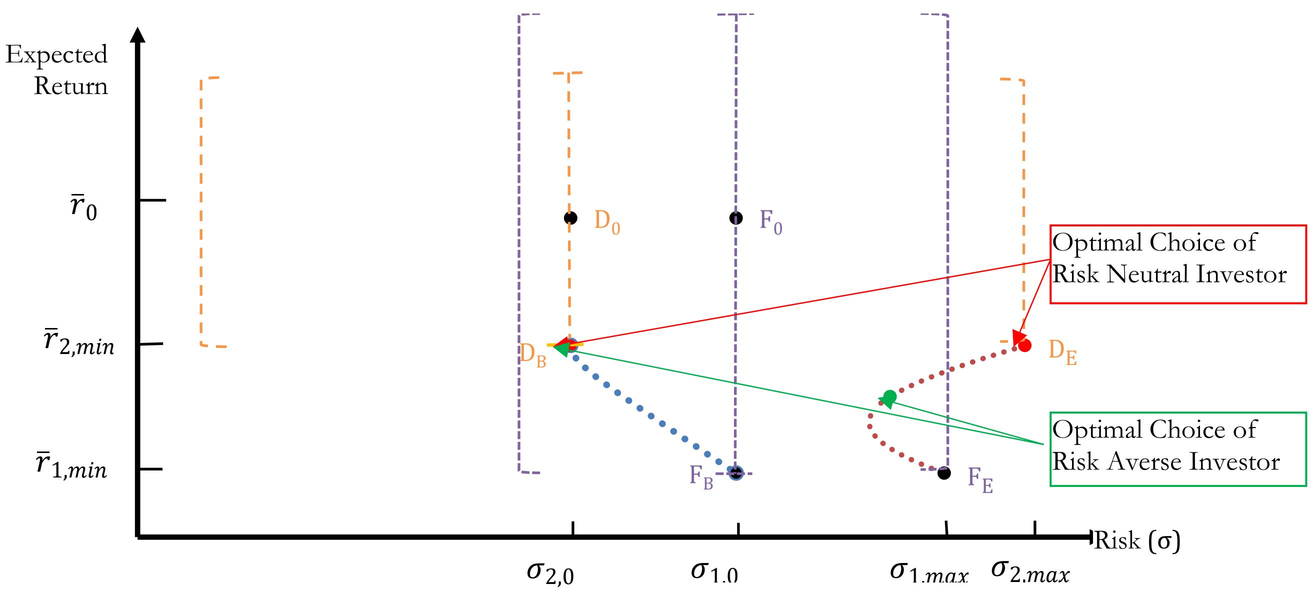
Figure 1. Optimal Portfolio Choice of Family Owners



Panel A: Non-Exiting Case ($\bar{r}_{1,min} > \bar{r}_{2,min}$)

F_A corresponds to the worse-case scenario return-volatility pair of the family firm stock ($\bar{r}_{1,min}, \sigma_{1,max}$), D_A corresponds to the worse-case-scenario return-volatility pair of the diversified portfolio ($\bar{r}_{2,min}, \sigma_{2,max}$). F_B and D_B correspond to the worse-case scenario of return-volatility pair of the family firm stock and the diversified portfolio, respectively, in the case where there is no ambiguity about return volatility.



Panel B: Exiting Case ($\bar{r}_{1,min} < \bar{r}_{2,min}$)

F_E corresponds to the worse-case scenario return-risk of family firm stock ($\bar{r}_{1,min}, \sigma_{1,max}$), D_E corresponds to the worse-case-scenario return-risk of the diversified portfolio ($\bar{r}_{2,min}, \sigma_{2,max}$). F_B and D_B correspond to the worse-case scenario return-risk of the family firm stock and the diversified portfolio, respectively, in the case where there is no ambiguity about risk of the returns.

Figure 2: Macro-Economic Uncertainty Index

This figure displays the three-month, and 12-month-ahead macro-economic uncertainty index developed by Jurado, Ludvigson, and Ng (2015). The data for the period July 1960-December 2017 are obtained from Sydney Ludvigson's website < https://www.sydneyludvigson.com/data-and-appendixes/ >.

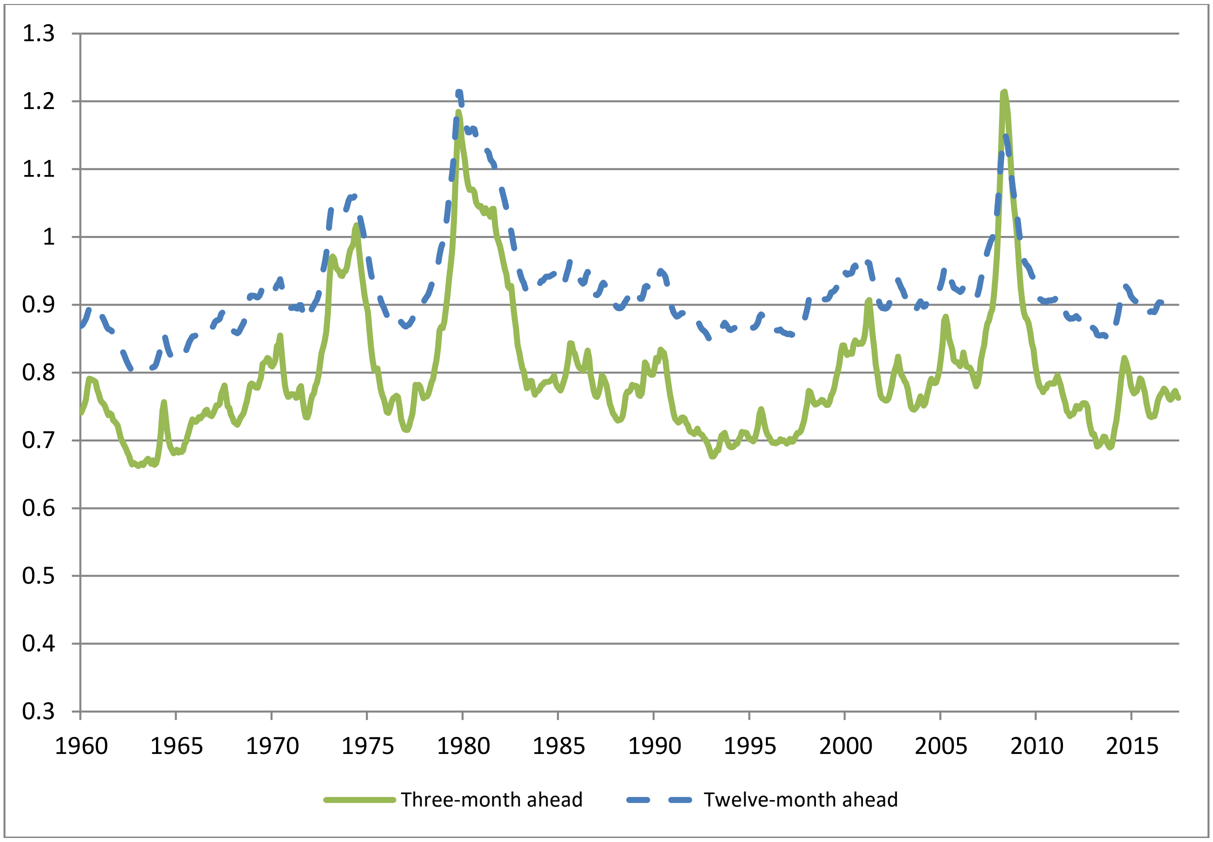


Table I – Variable Definitions

Debt Ratio – year-end long-term debt divided by total assets for each firm.

Equity issuance – the sum of common and preferred stock annual issuances divided by total assets.

Family Firm – binary variable that equals one when the family holds a five percent or larger ownership stake and zero otherwise.

Family Management – binary variable that equals one when either the founder or a founder's descendant serves as CEO of the firm; equals zero when an outside professional manager serves as CEO of the firm.

Family Ownership – the percent of common equity held by the family.

Family Risk Aversion – equals one when a female member of the family serves on the firm's board of directors and/or holds a 5% or larger equity stake.

Family Wealth – family fractional equity ownership stake multiplied by year-end firm total market value.

Growth Opportunities – market value of equity divided by total assets.

High Family Ownership – level of family equity stake greater than 20% of the firm's outstanding equity

Firm Uncertainty Beta - the coefficient estimate (β_1) of the JLN Macro Uncertainty index from the factor models.

Industry Uncertainty Beta - the coefficient estimate (β_1) of the JLN Macro Uncertainty index from the factor models.

Industry Dummy - equals one for each Fama-French industry group and zero otherwise.

Low Family Ownership - level of family equity stake greater than 5% but less than 20% of the firm's outstanding equity

Return on Assets$_{t-1}$ – earnings before interest, tax, depreciation and amortization (EBITDA) divided by total assets for the prior year-end for each firm.

Total Assets – year-end total assets for each firm.

Volatility – standard deviation of the growth of income before extraordinary items (ibq) for the prior 20-quarters for each firm.

Table II: Summary Statistics

Panel A: Monthly descriptive statistics for the Inputs to the Uncertainty Measure

This panel reports summary statistics of the dependent variable and main independent variables used to estimate uncertainty beta. The sample spans from January 2001 through December 2017, providing 204 monthly observations. The monthly risk-free rate of return, r_f, is taken from Kenneth French's website. The monthly stock return is taken from CRSP and excess return is calculated by deducting risk free rate from each firm's monthly stock return. Industry adjusted return is Fama-French monthly industry return for 48 industries minus monthly risk-free rate. Uncertainty is the 12-month ahead macro-economic uncertainty index from JLN (2015).

	N	*Mean*	*Median*	*Min*	*Max*	*SD*
Risk free rate	204	0.001	0.001	0.000	0.005	0.001
Return	204	0.008	0.010	-0.190	0.161	0.047
Excess return	204	0.007	0.009	-0.191	0.161	0.047
Ind. adjusted return	204	0.008	0.009	-0.433	0.800	0.075
Uncertainty	204	0.925	0.910	0.849	1.153	0.056

Panel B: Descriptive Statistics on the Firm Characteristics for the Full Sample

This panel provides summary statistics on annual data for the Russell 3000 non-financial, non-utility firms from January 2001 through December 2017, providing 22,426 firm-year observations on 2,154 unique firms. Firm and industry uncertainty beta is calculated using JLN (2015) 12-month ahead uncertainty index. Table I provides variable definitions.

	Mean	*Median*	*Min.*	*Max.*	*SD.*
Firm Uncertainty Beta					
w/simple model	0.003	-0.085	-1.879	2.306	0.665
w/market model	0.217	0.134	-1.655	2.486	0.651
w/Fama-French 5-factor model	0.115	0.054	-1.873	2.515	0.674
Industry Uncertainty Beta					
w/simple model	-0.061	-0.120	-0.723	1.034	0.336
w/market model	0.155	0.132	-0.477	0.857	0.250
w/Fama-French 5-factor model	0.073	0.047	-0.522	0.896	0.251
Family Ownership (%)	8.43	0	0	69.4	15.41
Total Assets ($M)	4,771	1,007	30.11	81981	11,587
Ln(total assets)	7.045	6.915	3.405	11.31	1.656
Debt Ratio	0.193	0.159	0	0.927	0.195
Return on Assets$_{t-1}$ (%)	11.24	12.77	-0.585	0.483	0.156
Tobin *Q*	2.00	1.611	0.702	7.676	1.234
Volatility	58.46	13.43	0.761	949.2	138.1
Equity Issuance/Assets	11.45	0.053	0.007	1.406	0.200

Panel C: Descriptive Statistics for Family and Non-family Firms

This panel provides summary statistics on annual data for the Russell 3000 non-financial, non-utility firms – segregated into family firms and non-family firms – from January 2001 through December 2017. Columns 1 and 2 use the full sample, providing 22,426 firm-year observations on 2,154 unique firms. Columns 3 and 4 using only family firms divided into low ownership (<=20% of outstanding equity) and high ownership (>20% of outstanding equity), providing 7,423 firm-year observations for 907 unique firms. *t*-statistics for difference of mean tests between non-family and family firms (low- and high family ownership) are provided in column 3 (6) and corrected for serial correlation by clustering on firm-level identifier. Table I provides variable definitions. ***, **, and * indicates significance at the 1%, 5%, and 10% levels, respectively.

	Firm Type			*Family Firms Only*		
	Non-family	*Family*	*t-stat*	*Low (<=20%)*	*High (>20%)*	*t-stat*
Observations	15,003	7,423		3,851	3,572	
Family Ownership (%)	0.356	24.75	35.87***	11.17	39.38	34.04***
Firm Uncertainty Beta	-0.008	0.026	2.2**	0.042	0.009	1.28
Industry Uncertainty Beta	--0.063	-0.059	.640	-5.233	-6.421	1.25
Total Assets ($M)	5,839	2,613	6.52***	2970	2228	1.18
Ln(total assets)	7.28	6.56	9.88***	6.63	6.49	1.4
Debt Ratio	0.203	0.173	3.72***	0.1667	0.1801	1.07
Return on Assets$_{t-1}$ (%)	11.09	11.52	0.78	9.89	13.28	4.19***
Tobin *Q*	2.03	1.91	2.64***	1.96	1.87	1.28
Volatility	69.5	36.15	6.28***	42.64	29.15	1.92*
Equity Issuance/Assets	11.36	11.63	0.37	13.01	10.15	2.38**

Panel D: Family exit sample

This panel provides annual summary statistics for families exiting their concentrated equity ownership stake (family-exit firms) and a matched sample of families maintaining their ownership stake (family firms). Family-exit firms are matched to family firms using coarsened exact matching (CEM) based on total assets, debt ratio, ROA, growth opportunity, equity issuance, and volatility. The process results in 292 unique firms (164 observations of family-exit firms matched to 164 family firm observations. Family-exit and family firms come from our base Russell 3000 non-financial, non-utility firms from January 2001 through December 2017. The variables are defined in Table I.

	All Firms	*Family-Exit Firms*	*Family Firms*	*t-statistic*
Family Ownership (%)	5.28	0.564	9.994	7.39***
Firm Uncertainty Beta	0.1357	0.183	0.088	1.16
Industry Uncertainty Beta	-0.021	-0.004	-0.037	0.84
Ln(Family Wealth)	1.519	0.936	2.107	5.16
Ln(total assets)	6.274	6.27	6.29	0.11
Debt Ratio	14.00	14.41	13.99	0.21
Return on Assets$_{t-1}$ (%)	8.54	8.60	8.67	0.04
Tobin *Q*	2.04	2.04	2.03	0.07
Volatility	48.33	48.90	45.31	0.22
Equity Issuance/Assets	18.30	18.21	18.30	0.03

Table III: Family Ownership Analysis

This table presents OLS regression results of family ownership on firm level and industry level uncertainty beta and firm characteristics. In columns 1-2, the dependent variable is *family ownership*, the percentage of ownership a family holds in the firm. In column 3-4, the dependent variable equals 0 for low-family ownership firms (>=5% and <=20%) and 1 for high family ownership firms (>=20%) . Uncertainty betas are estimated using JLN (2015) 12-month ahead uncertainty index. The variables are defined in Table I. Columns 1 and 2 include Russell 3000 non-financials and non-utility firms for the years 2001-2017. Columns 3 and 4 include only family firms. Columns 1 and 2 present coefficients from OLS regression with t-statistics in parentheses. Columns 3 and 4 present the odd ratios from logit regression with z-values in parentheses. ***,**, and * indicate significance at the 1%,5%, and 10% levels, respectively and corrected for serial correlation and heteroskedasticity by clustering on the firm-level identifier.

	OLS		*Logit*	
	Dependent variable = Family ownership (%)		*Dependent variable = (High Family Ownership=1; Low Family Ownership=0)*	
	(1)	*(2)*	*(3)*	*(4)*
Firm Uncertainty Beta	-0.004***	-	0.912***	-
	(2.79)		(2.67)	
Industry Uncertainty Beta	-	-0.024***	-	0.866***
		(5.93)		(2.02)
Ln (total assets)	-0.025***	-0.024***	0.811***	0.809***
	(28.65)	(26.10)	(8.68)	(8.73)
Debt Ratio	-0.041***	0.010*	1.886***	1.905***
	(7.72)	(1.91)	(4.89)	(4.97)
Return on Assets	0.060***	0.100***	7.277***	7.099***
	(7.45)	(12.37)	(9.47)	(9.36)
Growth opportunity	-0.003***	-0.007***	0.955*	0.957*
	(3.12)	(7.11)	(1.92)	(1.81)
Equity Issue	-0.052***	-0.053***	0.509***	0.497***
	(7.45)	(7.13)	(3.41)	(3.54)
Volatility	0.000***	0.000***	0.999	0.999
	(3.76)	(3.20)	(0.630)	(0.53)
Constant	0.292***	0.256***	0.3109***	3.100***
	(43.25)	(38.82)	(6.99)	(6.97)
Observations	22,426	22,426	7,423	7,423
Adj./*Pseudo* R^2	17.85	5.82	2.35	2.32
Year FE	Yes	Yes	Yes	Yes
Industry FE	Yes	No	Yes	No

Table IV Family Exit Analysis: Family Characteristics

Columns 1 through 8 report hazard ratios with a COX model of family exits and a coarsened exact matched (CEM) sample of family firms not exiting. Family exit is defined as binary variable which equals one if family ownership dropped to 3% and below in year *t* from 5% and above in year *(t-1)*. Uncertainty betas are estimated using JLN (2015) 12-month ahead uncertainty index. Variables are defined in Table I. z-values are reported in parentheses and are corrected for serial correlation and heteroskedasticity by clustering on firm level identifiers. ***, **,* indicate significance at the 1%,5%, and 10% levels, respectively.

	Dependent Variable = Family Exit (0 if no exit, 1 if an exit)							
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
Firm Uncertainty Beta	1.45***	1.40***	1.41***	1.32**	-	-	-	-
	(3.46)	(3.18)	(3.21)	(2.23)				
Industry Uncertainty Beta	-	-	-	-	1.72**	1.65**	1.70**	1.83**
					(2.37)	(2.21)	(2.32)	(2.48)
Family Management	-	0.29***	-	-	-	0.33***	-	-
		(-3.74)				(-3.35)		
Ln(Family Wealth)	-	-	0.88***	-	-	-	0.87***	-
			(-2.86)				(-2.97)	
Family Risk Aversion (women)	-	-	-	0.29**	-	-	-	0.29**
				(-2.12)				(-2.13)
Ln (total assets)	0.70***	0.68***	0.72***	0.70***	0.72***	0.72***	0.76***	0.69***
	(-3.50)	(-3.86)	(-3.19)	(-3.43)	(-3.43)	(-3.44)	(-2.90)	(-3.48)
Debt Ratio	1.83	1.78	1.68	1.75	1.56	1.55	1.56	1.78
	(1.19)	(1.14)	(1.03)	(0.95)	(0.91)	(0.87)	(0.89)	(0.97)
Return on Assets	1.18	0.87	1.26	1.01	1.78	1.14	1.67	1.20
	(0.24)	(-0.20)	(0.34)	(0.02)	(0.84)	(0.20)	(0.75)	(0.25)
Growth opportunity	1.04	1.06	1.06	1.03	0.91	0.92	0.92	0.99
	(0.49)	(0.81)	(0.80)	(0.27)	(-1.30)	(-1.17)	(-1.17)	(-0.04)
Equity Issue	2.16	1.88	2.10	1.20	3.16**	2.55*	3.07	1.18
	(1.48)	(1.23)	(1.43)	(0.30)	(2.35)	(1.91)	(2.30)	(0.28)
Volatility	1.00**	1.00**	1.00*	1.00***	1.00*	1.00*	1.00**	1.00***
	(2.07)	(2.11)	(1.93)	(2.60)	(1.78)	(1.65)	(1.43)	(2.63)
Observations	328	328	328	266	328	328	328	266

Table V: Family Ownership Analysis

This table presents OLS regression results of family ownership on firm-level and industry-level uncertainty betas and firm characteristics. In columns 1-2, the dependent variable is *family ownership*, the percentage of ownership a family holds in the firm. In column 3-4, the dependent variable equals 0 for low-family ownership firms (>=5% and <=20%) and 1 for high family ownership firms (>=20%). Uncertainty betas are estimated using JLN (2015) 12-month ahead uncertainty index. Panel A include the JLN factor and a market factor. Panel B includes the JLN factor and the Fama-French 5-factors. The variables are defined in Table I. Columns 1 and 2 include Russell 3000 non-financials and non-utility firms for the years 2001-2017. Columns 3 and 4 include only family firms. Columns 1 and 2 present coefficients from OLS regression with t-statistics in parentheses. Columns 3 and 4 present the odd ratios from logit regression with z-values in parentheses. ***,**, and * indicate significance at the 1%,5%, and 10% levels, respectively and corrected for serial correlation and heteroskedasticity by clustering on the firm-level identifier.

Panel A: *Uncertainty Beta calculated as estimate of* β_1 *in:* *Excess return* $=a + \beta_1 uncertainty + \beta_2 R_{Mt}\text{-}R_{Ft} + \varepsilon_{i,t}$				
	Dependent variable = Family ownership (%)		*Dependent variable = (High Family Ownership=1; Low Family Ownership=0)*	
	(1)	*(2)*	*(3)*	*(4)*
Firm Uncertainty Beta	-0.006***	-	0.873***	-
	(4.13)		(3.88)	
Industry Uncertainty Beta	-	-0.046***	-	0.752***
		(9.03)		(2.91)
Controls	Yes	Yes	Yes	Yes
Observations	22,426	22,426	7,423	7,423
Adj. R-squared (%)	17.88	6.01	2.42	2.36
Year FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	No
Panel B: *Uncertainty Beta calculated as estimate of* β_1 *in:* *Excess return* $= a + \beta_1 uncertainty + \beta_2 R_{Mt}\text{-}R_{Ft} + \beta_3 SMB + \beta_4 HML + \beta_5 RMW + \beta_6 CMA + \varepsilon_{i,t}$				
Firm Uncertainty Beta	-0.005***	-	0.880***	-
	(3.74)		(3.77)	
Industry Uncertainty Beta	-	-0.036***	-	0.753***
		(7.48)		(2.80)
Control	Yes	Yes	Yes	Yes
Observations	22,426	22,426	7,423	7,423
Adj./Pseudo R-squared (%)	17.87	5.90	2.42	2.35
Year FE	Yes	Yes	Yes	Yes
Industry FE	Yes	No	Yes	No

Table VI: Family Exit Analysis: Family Characteristics

Columns 1 through 8 report hazard ratios with a COX model of family exits and a coarsened exact matched sample of family firms not exiting. Family exit is defined as binary variable which equals one if family ownership dropped to 3% and below in year *t* from 5% and above in year *(t-1)*. Uncertainty betas are estimated using JLN (2015) 12-month ahead uncertainty index. Variables are defined in Table I. z-values are reported in in parentheses and are corrected for serial correlation and heteroskedasticity with firm clustering. ***, **,* indicate significance at the 1%,5%, and 10% levels, respectively.

Panel A: *Uncertainty Beta calculated as estimate of β_1 in:*
Excess return $=a + \beta_1 uncertainty + \beta_2 R_{Mt}\text{-}R_{Ft} + \varepsilon_{i,t}$

	Dependent Variable = Family Exit (0 if no exit, 1 if an exit)							
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
Firm Uncertainty Beta	1.71***	1.65***	1.67***	1.41***	-	-	-	-
	(4.29)	(4.06)	(4.10)	(2.62)				
Industry Uncertainty Beta	-	-	-	-	1.91**	2.00**	1.86**	2.17**
					(2.22)	(2.43)	(2.11)	(2.33)
Family Management	-	0.32***	-	-	-	0.29***	-	-
		(-3.74)				(-3.69)		
Ln(Family Wealth)	-	-	0.89**	-	-	-	0.88***	-
			(-2.66)				(-2.79)	
Family Risk Aversion (women)	-	-	-	0.28**	-	-	-	0.30**
				(-2.16)				(-2.08)

Panel B: *Uncertainty Beta calculated as estimate of β_1 in:*
Excess return $= a + \beta_1 uncertainty + \beta_2 R_{Mt}\text{-}R_{Ft} + \beta_3 SMB + \beta_4 HML + \beta_5 RMW + \beta_6 CMA + \varepsilon_{i,t}$

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
Firm Uncertainty Beta	1.26**	1.30**	1.28**	1.35**	-	-	-	-
	(2.03)	(2.27)	(2.18)	(2.40)				
Industry Uncertainty Beta	-	-	-	-	2.81***	3.11***	2.94***	2.24**
					(3.52)	(4.04)	(3.80)	(2.55)
Family Management	-	0.31***	-	-	-	0.29***	-	-
		(-3.51)				(-3.72)		
Ln(Family Wealth)	-	-	0.87***	-	-	-	0.86***	-
			(-3.09)				(-3.24)	
Family Risk Aversion (women)	-	-	-	0.28**	-	-	-	0.29**
				(-2.20)				(-2.11)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	328	328	328	266	328	328	328	266

Internet Appendix: Not for Publication

1: Model Implied Family Ownership

In part one, we present the implied share of family wealth invested in the family firm for various levels of relative ambiguity and expected returns in the model with ambiguity. The implied family-ownership is computed for different levels of ambiguity about the diversified portfolio relative to the family firm, and for three cases where the worse expected return of the family firm is the same, less, and more than that of the diversified portfolio. For comparison, we also present the implied ownership of a single stock in the model without ambiguity.

Table A.1.
Calibration of Stock Ownership

Model implied share of wealth invested in the single stock in models with and without ambiguity[23]. These estimates are computed in the benchmark model in Section 2. In the model without ambiguity, the true values of annual expected return ($\bar{r}_{2,0}$) and variance ($\sigma_{2,0}^2$) of the diversified portfolio are assumed to be 9% and 0.037, respectively. The true value of annual return variance ($\sigma_{1,0}^2$) of the single stock is assumed to 0.242, the correlation coefficient between diversified portfolio and single stock assumed to be 0.123 (see Elton and Gruber, 1977). In the model with ambiguity, the worse-case values of annual expected return ($\bar{r}_{2,min}$) and variance ($\sigma_{2,max}^2$) of the diversified portfolio are assumed to be same as the true values in the model without ambiguity.

Panel A: Model with Ambiguity			
Relative Ambiguity of Return Volatility ($\sigma_{2,max}^2/\sigma_{1,max}^2$)	Relative Ambiguity of Expected Returns $\bar{r}_{1,min} = \bar{r}_{2,min}$	$\bar{r}_{1,min} = 0.5\bar{r}_{2,min}$	$\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$
Infinity	100%	81%	>100%
4.00	82%	65%	98%
2.00	68%	54%	82%
1.00	50%	39%	61%
0.50	30%	22%	38%
0.25	13%	8%	17%
Panel B: Model without Ambiguity about Return Volatility			
Relative Return Volatility ($\sigma_{2,0}^2/\sigma_{1,0}^2$)	Relative Ambiguity of Expected Returns $\bar{r}_{1,min} = \bar{r}_{2,min}$	$\bar{r}_{1,min} = 0.5\bar{r}_{2,min}$	$\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$
0.15	3%	0%	6%

[23] The risk aversion and wealth level are calibrated such that 30% of wealth is invested in the risky portfolio and 70% invested in the safe asset, in Markowitz (1952) framework that contains a safe asset with return of 2% and a risky asset with expected return of 9%.

2: Family investment decision with Treasury Bills

We also consider the optimal portfolio choice problem faced by the family owners who choose to invest in three assets, the family's firm with return r_1, a diversified portfolio with return r_2 and Treasury Bills with return r_3. We use Treasury Bills as a proxy for the risk free asset with nonzero variance.[24]

Table A.2.
Summary Statistics of Return on Treasury Bonds and Market Index

Summary statistics of annual real returns on Treasuries and CRSP Stock Market Index. The returns are deflated using inflation rate computed from CPI. The returns on treasuries, CPI and market index are from CRSP dataset available on WRDS.

	Market Index	*Treasury Bonds/Bills*				
		30 year	*10 year*	*5 year*	*2 year*	*30 day*
Annual Return (%) 1942-2017	9.15	2.11	1.86	1.80	1.31	0.15
Annual Volatility (%) 1942-2017	18.50	13.51	9.71	7.10	5.20	3.55
Annual Return (%) 1995-2017	9.56	6.10	4.38	3.64	2.16	0.41
Annual Volatility (%) 1995-2017	20.19	17.61	8.95	5.93	3.59	2.11

Table A.2 shows that the average return and standard deviation of 30-day Treasury Bills is significantly lower than that of the market portfolio, and the standard deviation of the return on Treasury Bills is significantly different from zero. The standard deviation of longer term Treasury Bonds is much higher.

We assume that returns on all three assets, r_1, r_2 and r_3, are independent and normally distributed. To focus on the investment choice between the family firm and the market portfolio, we assume no ambiguity regarding the return on Treasury Bills. That is, we assume the true value of the mean and variance of returns on the family firm and the market portfolio, r_1 and r_2, are unknown to investors while the true value of the mean and variance of returns on Treasury Bills are known to all investors. The true value of the mean and variance of the return on asset *i*, r_i, is denoted as $(\bar{r}_{i,0}, \sigma_{i,0})$

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[24] 30-day Treasury Bills are commonly used as a proxy for the risk free asset in asset pricing and optimal portfolio choice literatures.

for i =1, 2, 3. Investors perceive that the mean and variance of r_1 and r_2 belong to a set of possible values $\Theta_1 = \{(r_{1,n}, \sigma_{1,m}), n = 1,2, \dots, N, m = 1,2, \dots, M\}$, and $\Theta_2 = \{(r_{2,n}, \sigma_{2,m}), n = 1,2, \dots, N, m = 1,2, \dots, M\}$, which contain the true values of mean and standard deviation ($\bar{r}_{i,0}, \sigma_{i,0}$), for i = 1, 2. That is, the true values $\bar{r}_{i,0}$ and $\sigma_{i,0}$ lie between $[\bar{r}_{i,min}, \bar{r}_{i,max}]$ and $[\sigma_{i,min}, \sigma_{i,max}]$, respectively, for i = 1, 2. We further assume that the mean return on Treasury Bills is smaller than the minimum mean returns on the family firm and the market portfolio, and the variance of return on Treasury Bills is smaller than the minimum variances of returns on the family firm and the market portfolio, that is,

$$\begin{gathered} \bar{r}_{1,min} > \bar{r}_3, \ \bar{r}_{2,min} > \bar{r}_3 \\ \sigma_{1,min} > \sigma_3, \ \sigma_{2,min} > \sigma_3 \end{gathered} \tag{A.1}$$

As in our benchmark model, we assume that family owners have private information regarding the fundamentals of the family firm, which allows them to reduce the ambiguity about the fundamentals of their own firms over time and through experience. However, the family owners cannot obtain material, non-public information about firms in the broad portfolio as they do not manage these other firms. In our setup, the reduction in the ambiguity is captured by the shrinkage in the range of all possible means and variances on the return on the family firm.

A.1.1 The family's investment decision

For each time period t, the decision problem of the family owners with CARA utility can be formulated as

$$\begin{gathered} \max_{\alpha_1,\alpha_2} \min_{\theta_1 \in \Theta_1, \theta_2 \in \Theta_2} E_t[-exp(\gamma W_{t+1})] \\ s.t. W_{t+1} = W_t[\alpha_1 r_{1,t+1} + \alpha_2 r_{2,t+1} + (1 - \alpha_1 - \alpha_2) r_{3,t+1}] \end{gathered} \tag{A.2}$$

where W_t is the wealth of the family at time t, $r_{1,t+1}$, $r_{2,t+1}$ and $r_{3,t+1}$ are the returns on the family firm, market portfolio and Treasury Bills at time t+1, respectively. α_1 and α_2 are the shares of wealth invested in the family firm and market portfolio, respectively. We allow for short sales in all the assets, and do not restrict α_1 and α_2 to be positive and less than one.

It can be easily shown that the decision problem of the family owners can be transformed as,

$$\begin{aligned} \max_{\alpha_1,\alpha_2} \min_{\bar{r}_{1,n_1}, \bar{r}_{2,n_2}, \sigma_{1,m_1}, \sigma_{2,m_2}} & \{[\alpha_1(\bar{r}_{1,n_1} - \bar{r}_3) + \alpha_2(\bar{r}_{2,n_2} - \bar{r}_3) + \bar{r}_3] \\ & - \frac{\gamma W_t}{2}[\alpha_1^2 \sigma_{1,m_1}^2 + \alpha_2^2 \sigma_{2,m_2}^2 + (1 - \alpha_1 - \alpha_2)^2 \sigma_3^2]\} \end{aligned} \tag{A.3}$$

Let's first examine the minimization problem of (A.3). Since the objective function in (A.3) is monotonically decreasing in σ_1^2 and σ_2^2 , the maximum possible value of variance is always chosen, that is

$$\sigma_1^* = \sigma_{1,max} \equiv \max_{m=1,2,..,M}\{\sigma_{1,m}\}$$

$$\sigma_2^* = \sigma_{2,max} \equiv \max_{m=1,2,..,M}\{\sigma_{2,m}\}$$

As long as the family is long in the assets, they only care about the minimum possible expected return of this asset. If the family shorts the asset, then the maximum possible mean return of this asset is chosen, that is,

$$(\bar{r}_1^*, \bar{r}_2^*) = \begin{cases} (\bar{r}_{1,min}, \bar{r}_{2,max}), & if\ \alpha_1 > 0\ and\ \alpha_2 < 0 \\ (\bar{r}_{1,min}, \bar{r}_{2,min}), & if\ \alpha_1 > 0\ and\ \alpha_2 > 0 \\ (\bar{r}_{1,max}, \bar{r}_{2,min}), & if\ \alpha_1 < 0\ and\ \alpha_2 > 0 \end{cases}$$

Assumption (A.1) implies that it is never optimal for the family owners to short both the family firm and the market portfolio at the same time. Given the solution to the minimization problem (A.3), the optimal share invested in the family business and the market portfolio can be characterized as following:

Case 1: If the following condition is satisfied,

$$(\bar{r}_{1,min} - \bar{r}_{2,max})\sigma_3^2 > (\bar{r}_{2,max} - \bar{r}_3)\sigma_{1,max}^2 + \gamma W_t \sigma_3^2 \sigma_{1,max}^2$$

then the family owners go long on the family stock and short the diversified portfolio

$$\alpha_1^* = \frac{(\bar{r}_{1,min} - \bar{r}_3)\sigma_{2,max}^2 + (\bar{r}_{1,min} - \bar{r}_{2,max})\sigma_3^2 + \gamma W_t \sigma_{2,max}^2 \sigma_3^2}{\gamma W_t(\sigma_{1,max}^2 \sigma_{2,max}^2 + (\sigma_{1,max}^2 + \sigma_{2,max}^2)\sigma_3^2)} > 0$$

$$\alpha_2^* = \frac{(\bar{r}_{2,max} - \bar{r}_3)\sigma_{1,max}^2 + (\bar{r}_{2,max} - \bar{r}_{1,min})\sigma_3^2 + \gamma W_t \sigma_{1,max}^2 \sigma_3^2}{\gamma W_t(\sigma_{1,max}^2 \sigma_{2,max}^2 + (\sigma_{1,max}^2 + \sigma_{2,max}^2)\sigma_3^2)} < 0$$

$$1 - \alpha_1^* - \alpha_2^* = \frac{-(\bar{r}_{1,min} - \bar{r}_3)\sigma_{2,max}^2 - (\bar{r}_{2,max} - \bar{r}_3)\sigma_{1,max}^2 + \gamma W_t \sigma_{1,max}^2 \sigma_{2,max}^2}{\gamma W_t(\sigma_{1,max}^2 \sigma_{2,max}^2 + (\sigma_{1,max}^2 + \sigma_{2,max}^2)\sigma_3^2)} \gtrless 0$$

This is the case when the family's private information allows them to significantly reduce the ambiguity of their family firm relative to the other firms, and the minimum expected return of family firm is much larger than the maximum possible expected return of the diversified portfolio (in excess of return on Treasury Bills), then the family chooses to invest as much as possible in their own firm. In this case, the share invested in the family firm decreases with the risk aversion and family wealth. However, we argue this is an extreme case and does not apply to the marginal family owners who may exit, as the family would never exit in this case. Furthermore, we argue this is a case rare in the reality, as it is extreme for the family to think the minimum risk adjusted expected return of the family firm is much larger than the maximum possible expected return of the diversified portfolio.

Case 2: If the following condition is satisfied,

$$-(\bar{r}_{1,min}-\bar{r}_3)\sigma_{2,max}^2-\gamma W_t\sigma_3^2\sigma_{2,max}^2<(\bar{r}_{1,min}-\bar{r}_{2,min})\sigma_3^2<(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+\gamma W_t\sigma_3^2\sigma_{1,max}^2,$$

then the family owners invest positive shares in both the family stock and the diversified portfolio

$$\alpha_1^*=\frac{(\bar{r}_{1,min}-\bar{r}_3)\sigma_{2,max}^2+(\bar{r}_{1,min}-\bar{r}_{2,min})\sigma_3^2+\gamma W_t\sigma_{2,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}>0$$

$$\alpha_2^*=\frac{(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+(\bar{r}_{2,min}-\bar{r}_{1,min})\sigma_3^2+\gamma W_t\sigma_{1,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}>0$$

$$1-\alpha_1^*-\alpha_2^*=\frac{-(\bar{r}_{1,min}-\bar{r}_3)\sigma_{2,max}^2-(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+\gamma W_t\sigma_{1,max}^2\sigma_{2,max}^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}\gtrless 0$$

In this case, the family owners still have less ambiguity about the expected return on their own firm relative to other firms. However, the private information advantage is not as large as it is in Case 1, so the family firm chooses to invest positive share of wealth in both the family firm and the diversified portfolio.

Case 3: If the following condition is satisfied,

$$(\bar{r}_{1,max}-\bar{r}_{2,min})\sigma_3^2<-(\bar{r}_{1,max}-\bar{r}_3)\sigma_{2,max}^2-\gamma W_t\sigma_3^2\sigma_{2,max}^2$$

then the family owners short the family stock and go long on the diversified portfolio

$$\alpha_1^*=\frac{(\bar{r}_{1,max}-\bar{r}_3)\sigma_{2,max}^2+(\bar{r}_{1,max}-\bar{r}_{2,min})\sigma_3^2+\gamma W_t\sigma_{2,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}<0$$

$$\alpha_2^*=\frac{(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+(\bar{r}_{2,min}-\bar{r}_{1,max})\sigma_3^2+\gamma W_t\sigma_{1,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}>0$$

$$1-\alpha_1^*-\alpha_2^*=\frac{-(\bar{r}_{1,max}-\bar{r}_3)\sigma_{2,max}^2-(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+\gamma W_t\sigma_{1,max}^2\sigma_{2,max}^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}\gtrless 0$$

This is scenario where the private information does not help to reduce the ambiguity of the family firm enough, so that the minimum possible expected return of the family business is less than the minimum possible expected return of the diversified portfolio.

Let us again focus on Case 2 where family owners invest positive shares in both the family firm and the diversified portfolio. In this case, we can rewrite the optimal investment share in the family firm as

$$\alpha_1^*=\frac{(\bar{r}_{1,min}-\bar{r}_3)/\sigma_3^2+(\bar{r}_{1,min}-\bar{r}_{2,min})/\sigma_{2,max}^2+\gamma W_t}{\gamma W_t(\sigma_{1,max}^2/\sigma_{2,max}^2+\sigma_{1,max}^2/\sigma_3^2+1)} \tag{A.4}$$

The optimal share depends on the Sharpe ratio of the family firm relative to the treasury bills as well as the market portfolio. When the family has less ambiguity about the variance and the expected return of the family firm, that is, the smaller are $\sigma^2_{1,max}/\sigma^2_{2,max}$ and $\sigma^2_{1,max}/\sigma^2_3$ or the larger are $(\bar{r}_{1,min} - \bar{r}_3)/\sigma^2_3$ and $(\bar{r}_{1,min} - \bar{r}_{2,min})/\sigma^2_{2,max}$, the more likely the family will continue to invest in the family firm. Thus, the key concern about risk adjusted return centers on the ambiguity of the variance rather than the level of the variance. Moreover, if the family has little information about the return volatility of the diversified portfolio (relative to their own firm), then they would put a very high upper bound on the perceived return volatility of this asset ($\sigma^2_{2,max}$ is high). This implies that they would invest little in the diversified portfolio. Suppose the family is extremely averse to ambiguity, or the relative ambiguity about the return volatility of the diversified portfolio gets extremely large, then the family would invest nothing in the diversified portfolio, regardless of their risk aversion or wealth level.

To study the relationship between the share invested in the family firm and the characteristics of the family such as risk aversion and wealth of the family, we found it is necessary to focus on the Case 2. Taking the derivative of the optimal share in family share (α^*_1) with respect to risk aversion (γ) and wealth (W_t), we have

$$\frac{\partial \alpha^*_1}{\partial(\gamma W_t)} = -\frac{(\bar{r}_{1,min}-\bar{r}_3)/\sigma^2_3+(\bar{r}_{1,min}-\bar{r}_{2,min})/\sigma^2_{2,max}}{(\gamma W_t)^2(\sigma^2_{1,max}/\sigma^2_{2,max}+\sigma^2_{1,max}/\sigma^2_3+1)} \tag{A.5}$$

If the family owners have less ambiguity regarding the family firm and think the return on their family firm is so good that the minimum return is sufficiently larger than that of the market portfolio such that the Sharpe ratio relative to the market portfolio is larger than the Sharpe ratio relative to the Treasury Bills, that is,

$$\frac{(\bar{r}_{1,min} - \bar{r}_{2,min})}{\sigma^2_{2,max}} > -\frac{(\bar{r}_{1,min} - \bar{r}_3)}{\sigma^2_3}$$

then the family will always invest positive share in the family business will not consider exiting. However, we are more interested in the case where

$$\frac{(\bar{r}_{1,min} - \bar{r}_{2,min})}{\sigma^2_{2,max}} < -\frac{(\bar{r}_{1,min} - \bar{r}_3)}{\sigma^2_3} < 0$$

The family thinks the minimum expected return of the family business is smaller than that of the market portfolio. This is the case that captures the marginal family investors who might consider the exit decision. In this case, (A.5) is positive and the more risk averse is the family or the more wealth the family has, the larger share of wealth is invested in the family firm, and the less likely for the family to exit, *ceteris paribus*. Hence, the predictions of our benchmark model still hold when the choice set of the family owners includes not only two risky assets with different levels of ambiguity but also a government bond that proxies for the relatively low risk asset without ambiguity.