A Theory of Reverse Asset Substitution*

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Abstract

This paper studies Reverse Asset Substitution (RAS), an agency problem in which banks place investment and borrowing restrictions on firms as part of a strategy to extract surplus from the firms over multiple periods. RAS arises for firms that cannot access public debt markets due to agency problems and cannot commit to a lending bank for a long relationship. RAS provides a constrained optimal lending solution to ensure banks can lend to firms despite this limited commitment problem. Under RAS, the restrictions imposed by banks commit the firms to having a prolonged lending relationship. RAS reduces a firm’s investment and leverage compared to the case in which firms can commit to a lending relationship with the bank.

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Bank financing is one of the most important sources of external financing for firms. We show that banks may restrict growth of the borrowing firms as part of a strategy to extract surplus from the firms over multiple periods. If banks do not share the upside of firms’ growth beyond avoidance of bankruptcy, they may restrict firm growth to benefit from a longer lending relationship. Sharpe (1990) and Rajan (1992) have discussed that banks may have bargaining power over borrowing firms since they are repositories of private information that other lenders do not have.\(^1\) This paper suggests that given this ability, a bank may supply credit and influence firm policy to maximize the bank’s profit over the length of the relationship with the firm. This results in reduced firm equity value and higher profits for the lender. We refer to this transfer of wealth from a firm’s equity holders to debt holders (i.e., banks) as Reverse Asset Substitution (RAS).

Reverse Asset Substitution is a solution to the limited commitment problem that a firm faces. Firms with large information asymmetry with capital markets may be able to finance their operations using bank financing. This is because when compared to arm’s length investors such as public debt markets, banks can provide relatively cheaper financing for firms facing information asymmetry and agency problems.\(^2\) However, as the firm grows, it cannot commit to returning to the bank for future financing needs, if the firm’s information gap with public debt markets reduces sufficiently. Thus, banks that provide the important service of supplying credit to firms while bridging their information asymmetry with public debt markets, create a commitment mechanism through RAS so that the firms get financed

\(^1\)Firms have “soft” information (such as the ability of the entrepreneur, quality of the product, etc.) that cannot be credibly communicated to other prospective lenders.

\(^2\)Literature suggests that banks reduce the agency costs of lending to firms in various ways. A bank screens prospective clients which reduces adverse selection (Diamond (1991)). Once a loan is provided, a bank can threaten to cutoff credit to incentivize the firm to make the right investments (Stiglitz and Weiss (1983)), thus reducing costs of moral hazard. This allows the bank to provide cheaper funds as it does not suffer from information asymmetry compared to arm’s length markets (James (1987)). Hoshi, Kashyap, and Scharfstein (1991), Petersen and Rajan (1994) and recently Bharath, Dahiya, Saunders, and Srinivasan (2009) show that availability of bank financing increases if banks have closer ties with the firms, supporting the information advantage hypothesis.
and the banks can maximize their profits. This is an optimal strategy for providing credit supply to firms facing limited commitment problem who would have otherwise not been financed in a constrained economy – where banks are not allowed to take equity stakes in the borrowing firms – as is the case in the United States.

RAS is in contrast to conventional asset substitution where in expectation, there is a transfer of wealth from debt holders to equity holders. Asset substitution is the agency problem when equity holders find it attractive to undertake risky projects. This is because if the project is successful, equity holders enjoy relatively larger gains, whereas if it is unsuccessful, debt holders suffer relatively larger losses. Debt holders impose investment and borrowing restrictions on the borrowing firm to mitigate the possibility of conventional asset substitution. In this paper, we show that the constraints may be imposed on firms to influence firm policy even when the borrowing firm is not prone to bankruptcy. This is because influencing firm policy in this manner may allow the lending bank to extract benefits of a lending relationship from the borrowing firm for a longer period. We argue that RAS has important quantitative implications, and leads to significant under-leverage in equilibrium, as borrowers trade-off financial flexibility for investment flexibility.\(^3\)

This paper investigates how a firm’s investment decisions are impacted by the lending bank’s objective to maximize its own profits. To answer this question, we model the profit maximization problem of a firm and its lending bank. Initially an entrepreneur, who owns the firm, does not have access to any source of capital but the bank. The lending bank has “soft” private information about the firm that public debt markets do not receive. In the future, after the firm has graduated above a certain threshold, access to public debt markets will be feasible as the importance of “soft” information dissipates. Thus, the owner-entrepreneur faces a tradeoff: if she obtains bank financing, she gets more financing than if

\(^3\)Empirically, Faulkender and Petersen (2006) have shown that firms that do not have access to public debt markets have significantly lower leverage even after controlling for demand-side factors.
she finances using internal cash flow of the firm. At the same time, the bank will try to ensure that the entrepreneur returns in the subsequent period so that the bank can extract surplus. Thus to maximize profits, the bank attempts to lock-in the borrowing firm in a profitable long-standing relationship. For this, the lending bank restricts the investment decisions of the firm. When the firm grows, and the cost of collection of information in proportion to loan size decreases, other borrowers compete and lending relationships become transactional and relatively less profitable for all lenders.\footnote{Empirically, Bharath, Dahiya, Saunders, and Srinivasan (2009) show that bank financing becomes transactional for larger firms.}

We start by showing two prominent features of bank covenants. These features are illustrated in Figures 1 and 2. Figure 1 shows the distribution of various covenants imposed by banks on borrowing firms. The dataset is all firms in DealScan database that are not distressed and thus can be considered healthy based on Altman’s Z-Score. The panel on the left shows that more covenants of certain types that constrain the balance sheet (Maximum Debt to Tangible Networth, Minimum Current Ratio, Minimum Quick Ratio) are placed on firms that are smaller, which is consistent with the theory that such firms may face higher chances of asset substitution and other agency conflicts. However, the figure to the right, shows that a different set of covenants that constrain net income (Maximum Capex, Max Debt to EBITDA, Min. Interest Coverage and Min. Fixed Charge Coverage) are imposed on firms that are in the mid-size range. This relatively higher weight of covenants on the firms in the middle range is at odds with the notion that banks are imposing covenants to reduce the possibility of asset substitution. Secondly, Figure 2 provides the incidence of Capital Expenditure covenants among firms based on their access to debt markets and shows Capital Expenditure covenants are popular over and beyond other covenants in firms that are closer to public debt market access.
These two features, (i) higher incidence of covenants restricting investment on mid-size firms and (ii) higher incidence of Capital Expenditures on firms closer to public debt market access are puzzling if asset substitution mitigation is the sole motive of banks imposing these covenants. Covenants dealing with possibility of repayment of loan in case of bankruptcy (where the limiting term is either current assets or tangible networth) are consistent with the view that banks impose covenants to protect themselves in case of bankruptcy, i.e., to mitigate asset substitution. However, the high incidence of Capital Expenditure covenants that restrict the dollar amount of capital expenditures by a firm, even if the firm is not close to bankruptcy, is at odds with the idea that a bank passively provides financing. It suggests a supply side effect of source of capital on investment and that all covenants are not imposed for the sole purpose of mitigating conventional asset substitution. This paper provides this other motive of lenders when entering financing contracts with firms.

This paper contributes to the literature on relationship banking. Mayer (1988) discusses the general inability of firms and creditors to commit to a mutually beneficial course of action since firms cannot commit to sharing future rents with their creditors. Fischer (1990) models the problem posed by competitive credit markets and suggests that a firm can commit to sharing future profits if the lending bank has information monopoly. Rajan (1992) shows that even the bank’s information monopoly may be insufficient to bind the firm when competition comes from arm’s-length markets. We argue that banks may restrict growth of the firm by limiting lending or investment to extend the duration of information monopoly. The firm may commit to this arrangement explicitly by accepting investment or debt covenants or implicitly by accepting lower amounts of financing from the bank in each period. The arrangement thus allows the firm to commit to sharing future rents with the bank, which is mutually beneficial and circumvents the problem pointed out in Mayer (1988).
Sharpe (1990), Hoshi, Kashyap, and Scharfstein (1991), Rajan (1992), Petersen and Rajan (1994), Petersen and Rajan (1995) and recently Bharath, Dahiya, Saunders, and Srinivasan (2009) show under various scenarios that relationship banking has its benefits and costs. In particular, Sharpe (1990) and Rajan (1992) show that banks may extract surplus from the borrowing firms. In this paper, we follow the aforementioned literature. We study the investment and financing restrictions (that may be explicit as covenants) a borrowing firm faces when it is in such a relationship. The bank does not want to lose this profitable exclusive relationship - an eventuality as the firm grows. Hence, the lending bank tightens the investment covenants on the borrowing firm to reduce the possibility of losing the relationship due to growth of the firm. The result is a firm that invests less and takes lower leverage even in the absence of the possibility of bankruptcy.

This paper contributes to the literature on capital structure, specifically the "under-leverage" puzzle. The motivation of firms to take on debt due to tax benefits is known since Modigliani and Miller (1963). Trade-off theory holds that the counterbalance to higher leverage is the increase in expected bankruptcy cost. Graham (2000) and subsequently Graham (2003) find that if a typical firm leveres up to the point at which the marginal tax benefits begin to decline, it could add 7.5% to firm value, after netting out the personal tax penalty. Bharath, Dahiya, Saunders, and Srinivasan (2009) show that if a creditor has a lending relationship with a firm, then such borrowing translates into a 10 to 17 basis points lowering of loan spreads. Given that firms can borrow cheaply and also get tax advantage on interest payments, why do firms not take more debt? What are the counterbalancing costs of debt - is it only the bankruptcy cost? We show that the agency problem between shareholders and debt-holders is a two-way street, and while asset substitution is a possibility, over-restriction is also observed in practice and is having an impact on firms’ willingness to take bank loans. This leads to a significant reduction in the leverage choice and value of a
firm as a going concern due to the cost of explicit or implicit restrictions on firm investment policy.

Section I models a lending relationship between a bank and a firm, where the firm has the ability to go to an arm’s length borrower at any point in the relationship. Section II characterizes the optimal lending contract and introduces RAS. Section III discusses the segmentation of lenders into banks, VCs and public debt providers based on project type and growth prospects. Section IV concludes.

I Setup

Consider a single owner-managed firm. The owner has a project idea and equity capital $E$ that only she can execute. In each period, the owner can invest an amount $I_0 \in [0, \bar{I}]$. Investment above $\bar{I}$ has no return and hence is inefficient. We assume $E < \bar{I}$, thus creating a need for external financing. The assets purchased can be liquidated in future periods for value $w$ per unit investment, where $0 \leq w \leq 1$.

There are two possible states at every date. The state can be good $G$ with probability $q$, and bad $B$ with probability $1 - q$. In the good state, the invested capital pays out $x$ per unit capital invested with certainty, $x > 1$. If state $B$ is realized, the project pays out $x$ per unit invested capital with probability $p_B$, and with remaining probability, all invested capital is lost. We assume $p_Bx < w < 1 < x$. If the project continues for the next period, the owner continues with retained capital $xI$ at date 1. She may refinance the capital using a different lender before each period. At the next date again, the state can be either good $G$ with probability $q$, or bad $B$ with probability $1 - q$. In each period, an exogenous quality parameter $\theta \in [0, 1]$ determine the probability $q(\theta)$ of the good state occurring. The quality remains the same between periods.
The timeline of the actions is as follows: (i) At time 0, the entrepreneur chooses a lender and asks for investment capital. The lender lends less than or the full amount sought by the borrower. Investment $I \in [0, \bar{I}]$ is made. (ii) State $(G/B)$ is realized. If lender is a bank, it can discontinue the project and liquidate invested capital for $wI$ if it wishes. (ii) On the next date, if the project continues, the project pays depending on state and probabilities. (iii) The owner can continue the project for the next period. The owner can refinance at this time and the cycle is repeated.

We now discuss the financiers, information setup and contract space to complete the description of the setup.

I.A Financiers

This is a risk-neutral economy and the risk-less interest rate is 0. The owner has equity capital $E$. The owner may choose to borrow more to invest in the project, or just invest what she has (autarky). There are two types of lenders that are competitive:

Banks

If a bank lends to a firm, it gains access to the internal records that a firm maintains. This bank thus becomes an “inside” bank. The bank collects “soft” information, that cannot be credibly communicated by the firm or the inside bank to outsiders. Banks have different costs of monitoring, thus some banks have a comparative advantage in lending to certain firms over others. This may be due to various reasons, such as geographic proximity of the bank to the firm or specialization of the bank in certain industries.
Arm’s-length investors

If such investors lend to the firm, they cannot examine the books of the firm. They lend and
the firm pays the debt back when the firm chooses to do so. There is no contact during the
intermediate period(s). This may be due to high private costs of monitoring as compared
to the banks, or because the arm’s length investors hold small shares of the whole loan,
resulting in a free-rider problem. A firm has to be larger than a certain size $E^{\text{min}}$ before it
can borrow from arm’s-length lenders, due to the assumption of higher costs of monitoring
for arm’s-length lenders.

I.B Information

The owner and the inside bank know the quality $\theta \in [0, 1]$ of the project at date 0. The
project quality $\theta$ maps into outcome probability $q(\theta)$ of good state $G$. An increasing $\theta$
corresponds to an increasing probability of a successful outcome i.e. $q_\theta(\theta) > 0$. The owner
also learns the state of the project, and chooses to continue the project or not. The inside
bank learns the state realized at the same time as the owner. The outside banks and arm’s
length investors observe public signals only, that are assumed to be uninformative.

I.C Contracts

The key feature of the contracts that drives the incentive incompatibility between banks and
firm equity holders is the limited commitment problem: firms cannot commit to returning
to the bank in future periods in successful states of the economy. This is because it will be
in the firm’s interests to go to external sources of financing at such a point. Hence banks are
apprehensive of the contracts that allow the firm to access financing now, grow at the optimal
rate for the equity holder and in the future access another source of financing depriving the
bank of future revenues.

We assume that contracts cannot be made contingent on the liquidation decision or state. Investment and state can be observed by the owner and inside bank only. We allow only debt contracts between the firm and bank or arm’s-length investor. This can be justified by appealing to a setup where state verification is costly. Without loss of generality, we consider debt contracts where a firm pays back interest and principal at the end of each period. We consider only one period loans. Allowing multiple period debt contracts from banks does not change the analysis, as we allow the bank and the firm to renegotiate the loan. Allowing multiple period arm’s-length debt contract does not change the analysis either as the firm always goes to the bank for a first period loan.

I.D  Bargaining Game

The bargaining game between owner and lender is as follows. We assume that the entrepreneur has exogenously determined bargaining power \( \mu \in [0,1] \) with respect to the lender. Thus, fraction \( \mu \) of the unallocated surplus of the project financed by debt \( B \) goes to the entrepreneur. The entrepreneur who obtains no debt receives a payoff of \( xE \) in case the project is successful. Hence, if the project is financed with debt of \( B \) from the lender in addition to equity \( E \) of the entrepreneur, then the entrepreneur receives an additional \( \mu(x - w)B \) where \( w \) is the recovery rate of debt. The lender receives \( (1 - \mu)(x - w)B + wB \) in case of success of the project. In case of failure, all equity is lost and the lender receives \( wB \).
II Contract Solution

We start with a numerical example of the optimal lending contract for the bank in section II.A before we characterize the first best solution in section II.B, and the contract solution for the arm’s length lender and the bank in sections II.C and II.D.

II.A A Simple Example

Let us assume there is a firm that finances itself with a bank from date 0 to some date $\tau$, and with arm’s length financing thereafter. The firm has a project that has a maximum investment capacity of $I_{\text{max}} = $100, and will return $x = 1.15$ if successful.

The owner has an equity of $E_0 = $10 and would like to borrow an amount of $B_0 = $90 from the bank to finance a project of $I_{\text{max}} = $100, where the break-even rate for the bank is $r_b = 0.05$. The bank observes the realized state. In the bad state, all equity is lost and the project is liquidated for $w = 0.8$ of borrowed assets and the lender keeps all the proceeds. In the good state, the contract does not obligate the bank to lend: this discretion vests the bank with the ability to hold up the owner, that it uses to extract surplus in return for allowing the owner to continue the project.

Solving the bargaining game in the case of successful project, the owner gets $\mu(x - w)B + xE = $17.8 while the lender gets $(1 - \mu)(x - w)B + wB = $97.2, where exogenous bargaining power $\mu = 0.20 \in [0, 1]$ is the share of unallocated surplus that the owner gets after bargaining.

For this example, we assume that if $E_{\tau+1}$ exceeds the minimum equity requirements $E_{\text{min}} = B_{\tau+1}/3$, then arm’s length financing is feasible at date $\tau$. Since $E_{\text{min}} + B_{\tau+1} = I_{\text{max}}$, when firm equity exceeds $E_{\text{min}} = $25, then arm’s length lenders are willing to lend to the firm.
If the project had a good state in previous period and if all the money the entrepreneur wanted was lent to her at date 0, the starting equity $E_1$ for the entrepreneur on date 1 has increased is:

$$E_1 = \mu(x - w)B_0 + xE_0 = $17.8$$

Again, the entrepreneur would like to borrow $B_1 = I_{max} - E_1 = $82.2. If the bank lends $B_1$ to the entrepreneur in this period and the project is successful, then the entrepreneur’s equity continues to increase on date 2:

$$E_2 = \mu(x - w)B_1 + xE_1 = $26.224$$

The owner at date 2 switches to arm’s length financing as now $E_2 > E_{min}$, and the bank loses the relationship with the owner’s firm. The total profits $L$ the bank made in this relationship is (given interest rate $r_b = 0.05$):

$$L = [(1 - \mu)(x - w) + w - 1][B_0 + e^{-0.05}B_1]$$

$$= 0.08 \ast ($90 + e^{-0.05}$82.2) = $13.455$$

Optimal Lending for Bank

As we have shown above, the total profits $L$ the bank made were $13.455$, which may not be the maximum profits the bank could have gotten from its relationship with the firm. The bank would like to choose the amount it lends in each period to maximize its total profits $L$ from the relationship.

The bank thus faces a tradeoff: if it restricts lending to the firm too much, the amount of surplus generated is reduced, reducing the extractable surplus along with it. Also because of a positive discount rate, the bank prefers to extract surplus earlier rather than later. On
the other hand, lending too much to the firm allows the firm to exit the relationship with
the bank faster, which would reduce the duration for which the bank can extract surplus.
The bank solves the profit maximization problem by solving for the amount it should lend
which when combined with entrepreneur’s equity in the firm decided the investment rate.
The investment rate in turn decides the total surplus generated by the firm while it is in
relationship with the bank. The surplus the bank can extract over the life-time of the
relationship depends in this manner on the amount lent by the bank.

The panel on the left in Figure 3 plots the total profits of the bank in the relationship
versus the amount of investment (as a percentage of maximum investment) made by the firm
in each period. The panel on the right plots the length of relationship versus the amount
of investment (as a percentage of maximum investment) made by the firm in each period.
The panel on the left in Figure 3 shows that bank makes maximum profit of $13.76 when
it allows the firm to make 91.5% of maximum investment in each period. The panel on the
right shows that the firm-bank relationship will last for 3 periods when the firm makes an
investment of 91.5%, thus extending the relationship by one period compared to when the
bank lends the maximum amount to the firm in each period. Figure 4 shows the evolution of
equity and bank loan over the relationship for the amount of lending optimal for the bank.
The horizontal line represents the threshold after which the firm at date 3 switches to arm’s
length financing.

[Insert Figures 3 and 4 Here]

In section II.B, we characterize the optimal lending contract for the bank. Subscript $b$
denotes bank, and $a$ denotes arm’s-length lender. The owner decides (i) what type of lender
to borrow from in each period, (ii) the amount of investment made, and (iii) whether to
continue or liquidate the project. The lender decides (i) the contract terms, that include
maximum amount he is willing to lend, (ii) whether he should renegotiate the deal or not.
II.B First-Best Solution

At each date, the owner should continue the project in the good state and close the project down in the bad state. The expected surplus is \( q(\theta)(x - 1)I - (1 - q(\theta))(1 - w)I \). The first term is the surplus if the good state is realized and \( I \) is invested at date 0, and the second term is the loss incurred in the bad state. Due to limited liability, the owner may not close the project in the bad state unless forced to do so. Since arm’s-length lenders cannot examine books or observe the realized state, they cannot close the project. If a bank is the lender, then it can close the project down if the bad state is realized.

Thus, if the owner borrows from a bank, the bank has more bargaining power allowing it to extract surplus. The bank realizes that the owner prefers the arm’s length lender, and would like to ensure that the owner comes back to the bank in future periods to borrow. The bank faces an inter-temporal tradeoff in terms of surplus that it can extract from the owner. If it finances the firm without any constraints, and the firm realizes the good state, then while the bank may extract a large surplus in the first period, the firm may also attain the capability to finance itself using arm’s length lenders in the next period and discontinues the relationship with the bank. In that case, the bank will get no surplus at all in future periods. The focus of this paper is what a bank does to maximize total surplus over the time it has a relationship with a firm, when it faces competition from an arm’s length lender. We also examine how the financing policy of such a bank impacts the investment policy of the borrowing firm.

We will first analyze the arm’s length contract and then bank contracts. We will derive conditions under which the owner goes to the arm’s length lender, and then finally the bank contract that maximizes bank’s surplus while restricting the firm owner from going to the arm’s length lender.
II.C Arm’s-Length Debt Contract

By assumption, the arm’s-length lender does not receive information about the actions or state realizations of the owner. The arm’s-length lender lends, and receives a payoff when the owner chooses to pay back the debt with interest. Let us first calculate the return expected by the arm’s length lender if he lends to the firm owner. The owner can borrow an amount $B_a$ at interest rate $r_a$, with interest paid every period. The owner invests all her equity $E$, and the borrowed capital, thus $I = B_a + E$. As discussed before, the total capital invested $I$ pays off $xI$ if the good state is realized, and $xI$ with probability $p_B$ if the bad state is realized. The owner always continues in intermediate periods even though it may be inefficient to do so, as the arm’s length lender cannot affect the owner’s decision to continue or liquidate.

For the contract to satisfy the feasibility constraint of the arm’s length lender, the return on the debt for the arm’s length lender must be less than the total expected payoff of the project:

$$B_a(1 + r_a) \leq x(q(\theta) + (1 - q(\theta))p_B)(E + B_a).$$

The Individual Rationality (IR) constraint of the arm’s length lender implies that the sum lent is less than the payoff in expectation:

$$B_a \leq (q(\theta) + (1 - q(\theta))p_B)B_a(1 + r_a).$$

In a competitive credit market, equation 2 holds with equality.

If the equity $E$ of the owner, and the quality of the project $\theta$ are sufficiently low, the return demanded for public debt may be so high that the feasibility constraint is not satisfied anymore. The minimum equity that the owner has to front to get a loan of size $B_a$ so that
the arm’s length lender’s IR constraint is satisfied is, for a given quality of project $\theta$:

$$E_{\text{min}}^a = B_a \left[ \frac{1}{(x[q(\theta) + (1-q(\theta))p_B])^2} - 1 \right].$$  \hspace{1cm} (3)

Follows from the IR constraint (equation 2) with an equality, the feasibility constraint (equation 1), and the identity $I = B_a + E$.

The lower bound on the quality of the project $\theta$ that requires investment $I_t$ that will be financed by the public debt markets with debt $B_t$ is:

$$\theta_{\text{min}} = g^{-1} \frac{B_t}{I_t} \text{ where } g(\theta) = (x[q(\theta) + (1-q(\theta))p_B])^2.$$  \hspace{1cm} (4)

The amount of equity needed is decreasing in the quality $\theta$ of project, which is intuitive. Moreover, as aversion to the bad state decreases with increasing $p_B$, the amount of equity needed for project financing is further reduced.

Next, we will analyze bank contracts.

II.D Bank Lending Contract

Let us assume there is a firm that finances itself with a bank from date 0 to $\tau \leq T$. The owner borrows $B_{b,t}$ from bank, where the discount rate for the bank is $\beta$. The bank observes the realized state. In the bad state, the project is liquidated for $wB_{b,t}$ that the bank captures. The owner retains the equity of the previous period if the project is stopped by the bank. This assumption is for simplicity, and does not affect the results qualitatively. In the good state, the contract does not obligate the bank to lend: this discretion vests the bank with the ability to hold up the owner, that it uses to extract surplus in return for allowing the owner to continue the project. Solving the bargaining game, the owner gets $\mu(x - w)B_{b,t} + xE_t$.
while the lender gets \((1 - \mu)(x - w)B_{b,t} + wB_{b,t}\), where exogenous bargaining power \(\mu \in [0, 1]\) is the share of unallocated surplus that the owner gets after bargaining.

In case the project is successful with probability \(q(\theta)\), the owner receives \(xE_t\) at time \(t + 1\) for his equity \(E_t\) in the firm, and in addition, also receives a share of the surplus generated by the capital \(B_t\) borrowed from the bank. The share \(\mu(x - w)B_t\) is determined by the bargaining game between the two parties, where \(w\) is the recovery rate of capital as noted earlier. The firm pays a fixed fee \(F\) for borrowing money from the bank in every period. In case the project does not success and the entrepreneur has borrowed capital from the bank, only fraction \(e\) of the invested equity remains. If all equity is lost, \(e = 0\). Thus, the law of motion for equity of the owner in a bank lending relationship is given as below:

\[
E_{t+1} = \begin{cases} \\
\mu(x - w)B_t + xE_t - F & \text{w. p. } q(\theta), \\
eE_t & \text{w. p. } 1 - q(\theta).
\end{cases}
\]  

(5)

Note that \(p_B > e\), i.e. in case the firm finances itself through all equity, the expected return on capital in case of failure \(p_B\) must be higher than \(e\). This is because there is no debt senior to equity in the firm and hence residual claim of equity holders will be higher in case of failure. Thus, on date \(\tau \leq T\), if the project has a good state in previous period, then the project yields \(xI_t\) for \(I_t\) units of capital invested. The owner gains some surplus even after the bank extracts a share. Thus, starting equity at date \(\tau\) is:

\[
E_{\tau+1} = \mu(x - w)B_{b,\tau} + xE_{\tau} - F
\]

(6)

If \(E_{\tau+1}\) exceeds the minimum equity requirements \(E_{\min}\) in equation 4, then arm’s length financing is feasible at date \(\tau\). Thus, the optimal strategy \(\phi\) of the borrower is to use bank financing until the last period when \(E < E_{\min}\) and arm’s length financing immediately after
that. This is under the assumption that once the threshold is crossed, even if the firm’s equity falls below $E_{\text{min}}$, arm’s length financing is available.

**Minimum Loan Size wanted by the Firm**

The individual rationality constraint of the firm (IR) determines a lower bound of financing it will accept in each period from any bank. The minimum debt $B_t$ that the firm would take from that bank should ensure that in expectation the firm has a higher equity with bank loan than without:

$$q(\theta)(\mu(x - w)B_t + xE_t) + (1 - q(\theta))eE_t - F \geq (q(\theta)x + (1 - q(\theta))p_B)E_t,$$

where $q(\theta)$ represents the probability of success of the project and $F$ represents the fixed cost of bank financing. The right hand side of the above equation represents the expected equity the firm will have in the next period in case it just invests its own equity. The left hand side represents the expected equity of the bank in case the firm borrows $B_{b,t}$ from the bank and invests it alongside its own equity. The above relation provides the minimum amount of debt $B_t$ that the bank has to provide for the firm’s Individual Rationality (IR) Constraint to be satisfied:

$$B^{\text{min}}_{b,t} = \frac{(1 - q(\theta))(p_B - e)E_t + F}{q(\theta)\mu(x - w)}$$

The minimum loan size per period ensures that in expectation the firm grows and eventually exceeds the threshold equity for accessing public debt markets. If the loan size offered by the bank is less than the minimum loan size acceptable to the firm, the firm continues with its own capital $E_t$ at time $t$. As equity of the owner $E_t$ in the firm increases, the minimum amount of debt $B_t$ sought also increases since the firm takes the risk of losing in expectation $p_B - e$ of the equity share if the entrepreneur borrows from the bank and allows the bank
seniority in capital structure.

**Bank Lending Problem**

The key point in our work is that a bank’s lending strategy should take into account its own profit maximization concerns. If the owner switches to arm’s length financing, then the bank cannot extract profits from the owner in the following period. Anticipating this, the bank chooses a contract $\sigma^*$ at date $t \leq \tau$ that maximizes its discounted payoff $S$ given borrower’s strategy $\phi$, which is to graduate to public debt market access as soon as possible by increasing the amount of equity to $E_\tau$.

The maximization problem of the bank is:

$$S^*_t(\sigma^*, \phi) = \max_{\sigma = (B_0)} \mathbb{E} \sum_{t=0}^{\tau} \beta^t \left[ q(\theta)((1 - \mu)(x - w) + w + (1 - q(\theta))w - 1]B_t + F | t < \tau \right]$$

where $B_t \geq \frac{(1 - q(\theta))p_bE_t + F}{\mu(x - w)q(\theta)}$

$$\mathbb{E} E_{t+1} = q(\theta)(\mu(x - w)B_t + xE_t) + (1 - q(\theta))eE_t - F$$

where after stopping time $\tau$ the bank extracts no surplus from the firm since the firm graduates to public debt markets at that time. The right hand side represents the discounted sum of the net surplus extracted in every period until stopping time $\tau$. With probability $q(\theta)$, the bank receives $((1 - \mu)(x - w) + w)B_{b,t}$ and with probability $(1 - q(\theta))$, the bank gets $wB_{b,t}$ for lent amount $B_{b,t}$. The profit of the bank is maximized over the length of relationship from time $t = 0$ to $t = \tau$. On each date, the minimum amount of debt the firm wants from the bank is determined based on equation 8. Thus, the decision of amount initially lent by the bank to the firm $B_0$ determines the expected evolution path of future debt and equity in the firm.
given by the two laws of motion (debt and equity) that constrain the profit maximization problem 9 (details in Appendix A). Thus, the bank strategy to maximize profits requires it to only choose initial amount lent to the firm.

**Optimal Loan Size and Duration of Relationship for Bank**

In this section, we determine the amount that the bank will lend $B_0$ to optimize the length of the relationship with the firm and to maximize bank’s profits. The total proceeds of a bank in a lending relationship can be written as:

$$S_t = \sum_{s=0}^{t} (\beta^s f B_s + F)$$

$$= \frac{f}{\beta} \frac{(a + b)^{t+1} - \beta}{a + b - \beta} B_0 + \left[ \frac{\beta^{t+1} g^{t+1} - 1}{\beta g - 1} - \frac{\beta^{t+1} - 1}{\beta - 1} f(a + b) + t \right] F$$

(11)

Thus, the total surplus extracted by a bank is the sum of a function of the amount lent $B_0$ at time 0 (which is the first term) and a function of the fixed fees obtained for lending in each period. While the first term increases as the initial amount and subsequent amount of capital lent increases, the second term increases if the length of the relationship increases.

Since if more debt is provided to a firm, it graduates faster to public debt markets, there is a tradeoff in terms of maximizing total bank surplus over the length of the relationship.

The optimal time period for the relationship of the bank with the firm is where the bank decides against lending any more at time $t = 0$ since it realizes that the expected period of relationship will then be reduced by a period, leading to a reduction in fixed fees $F$ for a period. At that point the marginal benefit of additional lending is offset by the marginal cost of the loss of fixed fee $\beta^t F$, giving us a relationship regarding the optimal duration $\tau$ of
the relationship based on first order condition with respect to $B_0$:

$$\frac{\partial S_t}{\partial B_0} - \frac{\partial \beta^t F}{\partial B_0} = 0,$$

Thus, the optimal duration $\tau$ of the relationship that the bank would like to have with the firm, in terms of initial debt $B_0$ (Detailed steps are in Appendix A):

$$\tau = \frac{\log(hK\beta(a + b - \beta)B_0^{\frac{B_0}{E_0}} + f\beta) - \log f(a + b)}{\log(a + b)}$$

The left panel of Figure 5 may be helpful in gaining an intuition regarding the implicit solution of optimal time $\tau$ of the relationship given above. The panel plots optimal length of the relationship $\tau$ as a function of project quality with other parameters fixed. The relationship between optimal duration of relationship and firm quality is non-monotonic. As the quality of the project rises, the bank seeks to prolong the relationship. However, after a certain level of quality ($q(\theta) \geq 0.6$ in this case), the discounted value of fixed fees becomes less important and the bank is willing to reduce the length of the relationship. In other words, the bank is trading off higher surplus in each period vs. length of relationship. After a certain level of quality (around $q(\theta) = 0.7$) however, the ability of the bank to extract surplus disappears and the lending relationship becomes transactional at this point as the bank has no ability to restrict the firm’s growth. The length of relationship at this point

5 where $a, b, c, f$ and $h$ are shorthand for (details in Appendix A):

$$\begin{align*}
a &= (1 - q(\theta))(p_B - e) \\
b &= q(\theta)x + (1 - q(\theta))e \\
c &= q(\theta)\mu(x - w) \\
f &= q(\theta)((1 - \mu)(x - w) + w) + (1 - q(\theta))w - 1 \\
h &= \frac{\beta^{t+1}g^{t+1} - 1}{\beta g - 1} - \frac{\beta^{t+1} - 1}{\beta - 1}f(a + b) + t
\end{align*}$$
drops to a single period relationship only.

[Insert Figure 5 Here]

Given that the minimum debt offered by the bank that the firm will accept in any period, is given by equation 8, we can obtain the initial debt $B_0$ that the bank will lend so as to keep the firm in a relationship until time $\tau$.

$$B_0 = \frac{1}{(a + b)^t} \left[ \frac{aE_\tau + F}{c} - (a + b)F_\tau \frac{g^t - 1}{g - 1} \right]$$  \hspace{1cm} (15)

The right panel of Figure 5 plots the initial debt offered by the bank in order to maximize its profits. As discussed above, the quality of the project rises, the bank reduces the initial loan size as it trades off obtaining fraction of surplus earlier for a longer relationship. After a certain level of quality (around $q(\theta) = 0.7$) however, the ability of the bank to extract surplus disappears as the firm only needs a small fraction of capital from the bank to grow out of the relationship. The lending relationship becomes transactional at this point as the bank has no ability to restrict the firm’s growth.

The reason that bank lends less than the amount sought by the firm is due to the inability of the firm to commit to return to the bank in the subsequent period. In the next section, we investigate the optimal contract in the presence of limited commitment that mitigates RAS.

III Optimal Financing in the presence of RAS

Section III.A discusses the choice of forms of financing based on the quality of projects $\theta$, growth prospects of the firm and equity of the owner. Section III.B derives the optimal contract in terms of debt and equity share that allows the bank to provide the necessary and
valuable service of bridging the information gap between firms and capital markets without restricting the growth of the firm through RAS. The key assumption remains that the firm is unable to commit to returning to the bank in future periods for financing, which causes banks to resort to RAS in the first place. However, allowing the bank to take an equity stake in the firm changes the tradeoff of the bank. It now also benefits from the growth of the firm and this reduces its incentive for prolonging the relationship to extract surplus. We derive the optimal amount of equity that the bank needs to own to ensure that the incentives of the bank and the firm are completely aligned. In this case, change in contract structure removes the agency problem of RAS.

III.A Financing Option based on Project Quality

In this section, we discuss the endogenous choice of forms of financing based on the quality of projects $\theta$, growth prospects of the firm and equity of the owner.

The three types of financiers that we consider are (i) Banks (ii) Public debt markets of investment grade and speculative grade (iii) Venture capital firms. The modeling assumptions regarding banks and public debt markets have been discussed in Section I.A. Like banks, if a VC lender finances a firm, it gains access to the internal records that a firm maintains. The VC firm thus becomes an “inside” party and collects “soft” information, that cannot be credibly communicated by the firm or the inside bank to outsiders. However, unlike a bank, a VC firm can also take an equity stake in the firm, thus bringing their financing contract closer to the optimal contract proposed in Section III.

The Individual Rationality (IR) constraint of the bank implies that there is a minimum quality of the project that the bank will finance (Details are in Appendix C):

$$q(\theta) = \frac{1 - w}{(1 - \mu)(x - w)}$$  \hspace{1cm} (16)
As the return $x$ of the project increases, or the bargaining power of the bank $(1 - \mu)$ increases, the quality of project that the bank is willing to finance decreases. Equation 4 in section II.C provided the minimum bound on the quality of the project for the arms length debt holder to finance the project, which in general is higher than the bound of banks due to the ability of banks to mitigate agency costs.

If the success probability of the project $q(\theta)$ is such that both those bounds are not satisfied then the only external financing left is an equity based financing through Venture Capital.

III.B Optimal Equity Share for the Bank

Let us assume that the firm has the choice of selling a share $\alpha$ of the firm at time $t = 0$ to the bank for $\alpha E_0$. The bank holds on to the equity until the end of relationship, when it sells the share for $\alpha E_\tau$. In this section, we characterize the contract to obtain $\alpha$ that removes RAS as the optimal strategy of the bank and restates a passive lending strategy as the optimal strategy.

The IR constraint of the firm determines the financing contract it will prefer in each period from any bank. If the firm has a choice to sell a portion of the equity to the bank and borrow the remaining amount it needs to invest $\bar{I}$, then it will choose to do so if the payoff is higher than in case of only borrowing $B^{RAS}$ that the bank will lend without owning any equity:

$$q(\theta)(\mu(x - w)(\bar{I} - E_t) + x(1 - \alpha)E_t) \geq q(\theta)(\mu(x - w)B^{RAS}_{b,t} + xE^{RAS}_t),$$  \hspace{1cm} (17)

where the firm owner sells fraction $\alpha$ of initial equity to the bank. The right hand side represents the expected equity of the firm in case the firm borrows $B_{b,t}$ from the bank and
invests it alongside its own equity. The bank is indifferent between this contract and the contract where it restricts investment if its payoff after obtaining equity is equal to or higher than the case where it can only enter a debt contract.

If the firm shares a portion of the equity with the bank, then the bank’s problem profit maximization problem is similar to equation 9 but includes one additional term $\beta^\tau \alpha E^\tau$ which is the payoff of the bank from the fraction $\alpha$ of equity owned by it at time $\tau$. Thus the profit maximization problem of the bank now becomes:

$$\max_{B,\tau} S_t = \max_{B,\tau} \mathbb{E} \sum_{t=0}^{\tau} \beta^t \left[ q(\theta)((1-\mu)(x-w) + w) + (1-q(\theta))w - 1 \right] B_t + F + \beta^\tau \alpha E^\tau$$  \hspace{1cm} (18)$$

where $B_t \geq \frac{(1-q(\theta))p_b E_t + F}{\mu(x-w)q(\theta)}$

$$\mathbb{E} E_{t+1} = q(\theta)(\mu(x-w)B_t + xE_t) + (1-q(\theta))eE_t - F$$  \hspace{1cm} (19)$$

In this case, the optimal time period for the relationship of the bank with the firm is the point where the bank trades off loss of fee $F$ for a period against marginal benefit of additional lending, and the loss of capital gains on its portion of equity.

The amount of equity that the firm shares with the bank in terms of initial debt and time of relationship (Details in Appendix B):

$$\alpha = \frac{f \frac{(a+b)^t+1-\beta}{\beta a+b-\beta} - Kh_{B_0} E_0}{(\beta^\tau - \beta^\tau+1) \left[ \frac{\xi}{a}(a + b)^t + \left[ \frac{(a+b)e g^t-1}{a g-1} - a \right] K_{B_0} E_0 \right]}$$  \hspace{1cm} (20)$$

When the quality of project is low, most of the surplus the bank obtains is from fixed fees. Providing equity to the bank in such cases helps align the incentives of the bank with the firm. Under certain conditions (details in Appendix B) optimal share of the bank in the firm
\( \alpha > 0 \). With the bank having a stake in the growth of the firm, the bank has less incentive in impeding the growth of the firm to extract fixed fees, and hence is willing to lend more as well.

[Insert Figure 6 Here]

Figure 6 seeks to obtain intuition regarding optimal share of equity to be owned by the bank to remove RAS. It plots the optimal equity held by the bank in order to maximize its profits. For very low quality projects, as discussed in Section III.A, bank financing is infeasible. However, as the quality of the project improves, the incentives of banks and firms can be aligned properly by allowing the bank to own an equity stake in the firm. As the quality of the firm keeps improving further, the bank’s interest can be aligned with the firm by sharing smaller fraction of equity with the firm. Beyond a certain point however, the relationship between the bank and the borrowing firm becomes transactional and no equity sharing is necessary in such a case.

Presently in U.S., banks are not allowed to take equity stakes in firms except in specific situations (An example case is when the firm is going through bankruptcy). Some large economies such as Japan and Germany on the other hand, allow banks to take substantial equity stakes in firms.

IV Conclusion

If a lender has private information about the borrower, then the lender can extract benefits from the borrower for a longer period if the borrower grows slowly. We use the term Reverse Asset Substitution (RAS) to express the agency problem where creditors benefit from slower growth of borrowing firms. Equity holders take this agency problem along with potential bankruptcy costs of debt into account when choosing firm leverage. RAS does not arise
for all firms. It arises for firms that cannot access public debt markets due to information asymmetry problems and cannot commit to a lending bank for a long relationship. RAS provides a constrained optimal lending solution to ensure banks can lend to firms despite this limited commitment problem. Under RAS, the restrictions imposed by banks commit the firms to having a prolonged lending relationship. RAS reduces a firm’s investment and leverage compared to the case in which firms can commit to a lending relationship with the bank.

Agency problems between equity holders and debt-holders are a two-way street, and while conventional asset substitution is a possibility, over restriction of investment by debt holders is also observed in practice and is having an impact on firms’ willingness to take bank loans. Faulkender and Petersen (2006) have shown that firms with access to public markets have 35% higher leverage. Thus, source of capital affects firm leverage. Chakraborty (2010) shows that bank financing reduces firm growth (11% lower PP&E) leading to a reduction of firm value by 23% compared to firms that have access to public debt markets. Hence, investment and financing restrictions by banks may have large quantitative importance for ex-ante leverage decision and investment of a firm.

If banks can also share the upside with firms, then the bank will not restrict the growth of the firm. Thus, if a bank holding company has a commercial bank arm that lends to firms when they are less successful, and also has an investment bank arm that will benefit from the firm growing and issuing public debt, then the agency problem discussed in this work will be less severe. Market developments after 2008, with major investment banks changing to bank holding companies, may have this unexpected and welcome effect of improving credit supply to firms.
References


Appendix

A Law of Motion of Bank Debt

We know that equity of the firm evolves according to the following law of motion:

\[ E_{t+1} = \begin{cases} 
\mu(x - w)B_t + xE_t - F & \text{w. p. } q(\theta), \\
e E_t & \text{w. p. } 1 - q(\theta).
\end{cases} \quad (21) \]

where for notational convenience we represent the fixed cost per period with \( F \):

\[ F = \frac{1}{2} K \frac{B_0^2}{E_0} \quad (22) \]

The expected equity in the next period as:

\[ \mathbb{E} E_{t+1} = q(\theta)(\mu(x - w)B_t + xE_t) + (1 - q(\theta))eE_t - F \quad (23) \]

The minimum debt \( B_t \) that the firm would take from that bank should ensure that in expectation the firm has a higher equity with bank loan than without:

\[ q(\theta)(\mu(x - w)B_t + xE_t) + (1 - q(\theta))eE_t - F \geq (q(\theta)x + (1 - q(\theta))p_B)E_t, \quad (24) \]

where \( q(\theta) \) represents the probability of success of the project and \( F \) represents the fixed cost of bank financing. The above relation provides the minimum amount of debt \( B_t \) that the bank has to provide for the firm’s Individual Rationality (IR) Constraint to be satisfied:

\[ B_t = \frac{(1 - q(\theta))(p_B - e)E_t + F}{q(\theta)\mu(x - w)} \quad (25) \]

which yields a recursive relationship of \( B_t \):

\[
B_t = \frac{(1 - q(\theta))(p_B - e)}{q(\theta)\mu(x - w)} E_t + \frac{1}{q(\theta)\mu(x - w)} F \\
= \frac{(1 - q(\theta))(p_B - e)}{q(\theta)\mu(x - w)} (q(\theta)(\mu(x - w)B_{t-1} + xE_{t-1}) + (1 - q(\theta))eE_{t-1} - F) + \frac{1}{q(\theta)\mu(x - w)} F, 
\]
which when solved further yields:

\[
B_t = (1 - q(\theta))(p_B - e)B_{t-1} + \frac{(1 - q(\theta))(p_B - e)q(\theta)x + (1 - q(\theta))e}{q(\theta)\mu(x - w)} E_{t-1} + \frac{1 - (1 - q(\theta))(p_B - e)}{q(\theta)\mu(x - w)} F
\]

\[
= (1 - q(\theta))(p_B - e)\left[B_{t-1} + (q(\theta)x + (1 - q(\theta))e)B_{t-2}\right]
+ \frac{(1 - q(\theta))(p_B - e)q(\theta)x + (1 - q(\theta))e^2}{q(\theta)\mu(x - w)} E_{t-2}
+ \frac{1 - (1 - q(\theta))(p_B - e)(1 + q(\theta)x + (1 - q(\theta))e)}{q(\theta)\mu(x - w)} F
\]  

Using the following shorthand:

\[
a = (1 - q(\theta))(p_B - e)
b = q(\theta)x + (1 - q(\theta))e
c = q(\theta)\mu(x - w)
f = q(\theta)((1 - \mu)(x - w) + w) + (1 - q(\theta))w - 1
\]

we can rewrite the recursive relationship of \(B_t\) as:

\[
B_t = aB_{t-1} + abB_{t-2} + \frac{ab^2}{c} E_{t-2} + \frac{1 - a(1 + b)F}{c}
\]

\[
= \sum_{s=1}^t ab^{s-1}B_{t-s} + \frac{ab^t}{c} E_0 + \frac{1 - \sum_{s=1}^t a(1 + b)^{s-1}F}{c}
\]  

Taking difference between \(B_t\) and \(bB_{t-1}\), we get \(B_t\) only in terms of \(B_{t-1}\), we get:

\[
B_t - bB_{t-1} = \sum_{s=1}^t ab^{s-1}B_{t-s} + \frac{ab^t}{c} E_0 + \frac{1 - \sum_{s=1}^t a(1 + b)^{s-1}F}{c}
- \sum_{s=1}^{t-1} ab^s B_{t-s} + \frac{ab^t}{c} E_0 + \frac{1 - \sum_{s=1}^{t-1} ab(1 + b)^{s-1}F}{c}
\]

\[
= aB_{t-1} + \frac{(1 - b)(1 - \sum_{s=1}^{t-1} ab(1 + b)^{s-1})}{c} F
\]

\[
B_t = (a + b)B_{t-1} + \frac{(1 - b)(1 - a((1 + b)^t - 1))}{c} F
\]
Thus we can represent $B_t$ in terms of initial debt $B_0$ as:

$$B_t = (a + b)^t B_0 + (a + b) F g^t - 1 \over g - 1$$

$$g = (1 - b)(1 - a((1 + b)^t - 1))$$  \hspace{1cm} (31)

The maximization problem of the bank is:

$$\max S_t = \max_{B, \tau} \mathbb{E} \sum_{t=0}^{\tau} \beta^t \left[ q(\theta)((1 - \mu)(x - w) + w) + (1 - q(\theta))w - 1\right] B_t + F \right]$$  \hspace{1cm} (32)

where $B_t \geq (1 - q(\theta))p B_t + F$  \hspace{1cm} (33)

Thus, the total proceeds of the bank can be written as:

$$S_t = \sum_{s=0}^{t} (\beta^s f B_s + F)$$

$$= \sum_{s=0}^{t} \beta^s (f [(a + b)^s B_0 + (a + b) F g^s - 1 \over g - 1] + F)$$

$$= f \over \beta \left[ (a + b)^{t+1} - \beta \right] B_0 + \left[ {\beta^{t+1} g^{t+1} - 1 \over \beta g - 1} - {\beta^{t+1} - 1 \over \beta - 1} f(a + b) + {1 - \beta^t \over 1 - \beta} F \right]$$  \hspace{1cm} (34)

Thus, the optimal time period for the relationship of the bank with the firm is where the bank decides against lending any more at time $t = 0$ since it realizes that the expected period of relationship will then be reduced by a period, leading to a reduction in fixed fees $F$ for a period. At that point the marginal benefit of additional lending is offset by marginal cost of loss of fixed fee $F$:

$$\frac{\partial S_t}{\partial B_0} - \frac{\partial \beta^t F}{\partial B_0} = f \over \beta \left[ (a + b)^{t+1} - \beta \right] B_0 + \left[ {\beta^{t+1} g^{t+1} - 1 \over \beta g - 1} - {\beta^{t+1} - 1 \over \beta - 1} f(a + b) + {1 - \beta^t \over 1 - \beta} F \right] = 0, \hspace{1cm} (35)$$
where $h$ is shorthand for:

$$
h(t) = \beta^t \left[ \frac{\beta^{t+1}g_{t+1} - 1}{\beta g - 1} - \frac{\beta^{t+1} - 1}{\beta - 1} f(a + b) + \frac{1 - \beta^t}{1 - \beta} \right] \quad (36)$$

Thus we get an implicit relationship between the optimal duration $\tau$ of the relationship that the bank would like to have with the firm and the initial debt $B_0$ that the bank would lend to maximize profits:

$$
\tau = \frac{\log(h(t)K\beta(a + b - \beta)\frac{B_0}{E_0} + f\beta) - \log f(a + b)}{\log(a + b)} \quad (37)
$$

Given that the minimum debt offered by the bank that the firm will accept in any period, is given by equation 8, the debt and equity of the firm at the time of end of relationship have the following relationship:

$$
B_{\tau} = \frac{aE_{\tau} + F}{c} \\
(a + b)^t B_0 + (a + b)Fg^t - 1 \quad = \quad \frac{aE_{\tau} + F}{c} \\
(38)
$$

which yields the initial debt $B_0$ that the bank will lend so as to keep the firm in a relationship until time $\tau$, in terms of the equity at the time when the firm switches to arm’s length lending relationship:

$$
B_0 = \frac{1}{(a + b)^t} \left[ \frac{aE_{\tau} + F}{c} - (a + b)F \frac{g^t - 1}{g - 1} \right] \quad (39)
$$

### B Optimal Bank Financing Contract

If the bank owns a fraction $\alpha$ of the equity of the firm, then the maximization problem of the bank becomes:

$$
\max_{S_t} S_t = \max_{B_{\tau}} \mathbb{E} \sum_{t=0}^{\tau} \beta^t \left[ q(\theta)((1 - \mu)(x - w) + w) + (1 - q(\theta))w - 1 \right] B_t + F + \beta^\tau \alpha E_{\tau} \quad (40)
$$

where $B_t \geq \frac{(1 - q(\theta))p_b E_t + F}{\mu(x - w)q(\theta)}$

$$
\mathbb{E} E_{t+1} = q(\theta)(\mu(x - w)B_t + xE_t) + (1 - q(\theta))eE_t - F \quad (41)
$$
Using relationship 39, we get the marginal increase in equity value at the stopping time \( \tau \) w.r.t. an additional dollar lent:

\[
\frac{\partial E_\tau}{\partial B_0} = \frac{c}{a} (a + b)^t + \left[ \frac{(a + b)c}{a} g^t - 1 - a \right] K \frac{B_0}{E_0} \tag{42}
\]

Since the bank will have to wait an additional period to sell the equity if it is in the relationship, then the additional loss in this case is \( \alpha (\beta^\tau - \beta^{\tau+1}) \frac{\partial E_\tau}{\partial B_0} \).

The optimal time period for the relationship of the bank with the firm is where the bank decides against lending any more at time \( t = 0 \) since it realizes that the expected period of relationship will then be reduced by a period, leading to a reduction in fixed fees \( F \) for a period. At that point the marginal benefit of additional lending, and the loss of capital gains is offset by marginal cost of loss of fixed fee \( F \):

\[
\frac{\partial S_t}{\partial B_0} - \frac{\partial \beta^t F}{\partial B_0} = \frac{f (a + b)^{t+1} - \beta}{a + b - \beta} - \alpha (\beta^\tau - \beta^{\tau+1}) \left[ \frac{c}{a} (a + b)^t + \left[ \frac{(a + b)c}{a} g^t - 1 - a \right] K \frac{B_0}{E_0} \right] - Kh \frac{B_0}{E_0} = 0, \tag{43}
\]

which yields the amount of equity that the firm shares with the bank in terms of initial debt and time of relationship:

\[
\alpha = \frac{\frac{f (a + b)^{t+1} - \beta}{a + b - \beta} - Kh \frac{B_0}{E_0}}{(\beta^\tau - \beta^{\tau+1}) \left[ \frac{c}{a} (a + b)^t + \left[ \frac{(a + b)c}{a} g^t - 1 - a \right] K \frac{B_0}{E_0} \right]} \tag{44}
\]

Under the condition that the returns of investment \( x \) are higher than liquidation value of the firm \( p_B \) in the bad state of the world i.e. \( x > p_B \), which is an assumption introduced in section I, we have \( a + b > 1 \). For a long enough duration of relationship \( \tau \) optimal share of the bank in the firm \( \alpha > 0 \).

### C  Financing Option based on Project Quality

The Individual Rationality (IR) constraint of the bank implies that the sum lent is less than the payoff in expectation:

\[
B_b \leq (((1 - \mu)(x - w) + w)q(\theta) + (1 - q(\theta))w)B_b, \tag{45}
\]
which yields the minimum quality of the project that the bank will finance:

\[ q(\theta) = \frac{1 - w}{(1 - \mu)(x - w)} \]  \hspace{1cm} (46)
This figure provides the distribution of covenants among firms divided into deciles based on their sales. The dataset is all firms in DealScan database. The figure to the right shows that more covenants of certain types (Maximum Debt to Tangible Networth, Minimum Current Ratio, Minimum Quick Ratio) are placed on firms that are smaller. However, the figure to the left, shows that more covenants of certain types (Maximum Capex, Max Debt to EBITDA, Min. Interest Coverage and Min. Fixed Charge Coverage) are imposed in firms that are in the middle size range. The difference in the relative distribution of covenants cannot be explained if all covenants play the same role of just reducing asset substitution.
The figure provides the incidence of Maximum Capital Expenditure covenants among firms based on their access to debt markets. The dataset is all firms in DealScan database that eventually received a credit rating. The vertical axis shows the number of Capital Expenditure covenants scaled by the number of total covenants. The horizontal axis is time in years. Time 0 is when a firm gets access to public debt markets. The relative frequency of Capital Expenditure covenants are adjusted by total number of covenants in each bin, to see differential importance of Capital Expenditure covenants over other covenants.
The figure on the left plots the total profits of the bank in the relationship versus the amount of investment (as a percentage of maximum investment) made by the firm in each period. The figure on the right plots the length of relationship versus the amount of investment (as a percentage of maximum investment) made by the firm in each period. The firm has a project that has a maximum investment capacity of \( I_{\text{max}} = 100 \), and will return \( x = 1.15 \) if successful. The owner has an equity of \( E = 10 \) and would like from the bank to finance the project. The break-even rate for the bank is \( r_b = 0.05 \). The bank observes the realized state. In the bad state, all equity is lost and the project is liquidated for \( w = 0.8 \) of borrowed assets and the lender keeps all the proceeds. In case of successful project, the owner gets \( \mu (x - w) B + w B \) while the lender gets \((1 - \mu)(x - w)B + wB\), where exogenous bargaining power \( \mu = 0.20 \in [0, 1] \) is the share of unallocated surplus that the owner gets after bargaining.
Figure 4: Evolution of Equity and Debt over the Time of Relationship

The figure plots the evolution of equity of firm and loan borrowed during the relationship with the bank. The firm has a project that has a maximum investment capacity of $I_{\text{max}} = 100$, and will return $x = 1.15$ if successful. The owner has an equity of $E = 10$ and would like from the bank to finance the project. The break-even rate for the bank is $r_b = 0.05$. The bank observes the realized state. In the bad state, all equity is lost and the project is liquidated for $w = 0.8$ of borrowed assets and the lender keeps all the proceeds. In case of successful project, the owner gets $\mu(x-w)B + xE$ while the lender gets $(1-\mu)(x-w)B + wB$, where exogenous bargaining power $\mu = 0.20 \in [0,1]$ is the share of unallocated surplus that the owner gets after bargaining.
Figure 5: Initial Loan and Time of Relationship as a function of Project Quality

The figure plots the initial debt (left) offered by the bank in order to maximize its profits, and the length of the relationship (right). The parameters are as follows: $p_b = 0.35; x = 1.25; e = 0; \mu = 0.25; w = 0.75; F = 0; \beta = 0.1; \frac{E_t}{10}; E_t = 10; E_0 = E_t / 10.$
Figure 6: Optimal Equity of Bank in the Borrowing Firm

The figure plots the optimal equity held by the bank in order to maximize its profits. The parameters are as follows: $p_b = 0.35; x = 1.25; e = 0; \mu = 0.25; w = 0.75; F = 0.1; \beta = 0.75; E_r = 10; E_1 = E_r/10$. 